Quantum integration of decay processes at high-energy colliders

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There is a growing interest in developing innovative strategies based on quantum computing to address traditionally difficult problems across various fields. One area set to fundamentally benefit from these technologies is the study of nature at its most fundamental level [1].

Quantum Field Theory (QFT) is an exceptionally successful theoretical framework for understanding nature at sub-atomic scales. Theoretical predictions for scattering and decay processes at high-energy colliders are based on Feynman diagrams as a graphical representation to encode the perturbative QFT solution from which we can calculate the probability of an event occurring. Feynman diagrams translate into complex mathematical expressions in the form of integrals. These integrals are difficult to compute due to their multidimensional nature, their dependence on multiple scales, and the presence of singularities, which often require tedious techniques to make them mathematically well-defined. Traditional numerical methods are also computationally intensive. This study aims to exploit the potential of quantum algorithms for the efficient computation of loop Feynman integrals, and specifically of decay rates at second order in perturbation theory or next-to-leading order (NLO). Although our central motivation is particle physics, this study is of interest in any other field where multidimensional integrals are required.

The intersection of quantum algorithms and particle physics is supported by several contributions, including lattice gauge theories [2, 3], jet identification and clustering [4–6], determining parton densities (PDFs) [7], simulating parton showers [8], detecting anomalies [9, 10], integrating elementary particle processes [11], and bootstrapping the causal structure of multi-loop Feynman diagrams [12–14]. The range of applications continues to expand rapidly, demonstrating the suitability of quantum algorithms for a wide array of purposes.

Indeed, we have recently proposed a new quantum algorithm, dubbed Quantum Fourier Iterative Amplitude Estimation (QFIAE) [15], for estimating multidimensional integrals. QFIAE leverages the fact that one can build a Quantum Neural Network (QNN) to represent a Fourier series [16]. In particular, QFIAE involves training a QNN to learn the target function, then extracting the associated Fourier coefficients and finally integrating each trigonometric component using Iterative Quantum Amplitude Estimation (IQAE) [17], an efficient version of Quantum Amplitude Estimation (QAE) [18] that does not rely on Quantum Phase Estimation (QPE) [19], making it possible to run it on hardware. Consequently, this method constitutes a comprehensive quantum algorithm that offers a viable strategy to maintain the quadratic speedup on the number of queries to the probability distribution function offered by the underlying Amplitude Amplification algorithm in QAE. The workflow of QFIAE is illustrated in Fig. 1.

QFIAE provides a solution for the challenges encountered by QAE. By using IQAE to avoid the necessity of QPE and by encoding cosines and sines functions in the initial state as a result of the Fourier series decomposition, QFIAE provides a clear workflow on how to apply it to any integration problem, which differs from QAE which faces a different encoding problem for every function that one wants to integrate.

In the context of particle physics, QFIAE has been applied to estimate Feynman loop integrals with high accuracy [21]. In this work, we go one step further and apply QFIAE to compute physical observables following the method described in [22–24], which provides a unified framework for encoding all the perturbative quantum fluctuations from a vacuum amplitude that acts as a kernel. In particular,

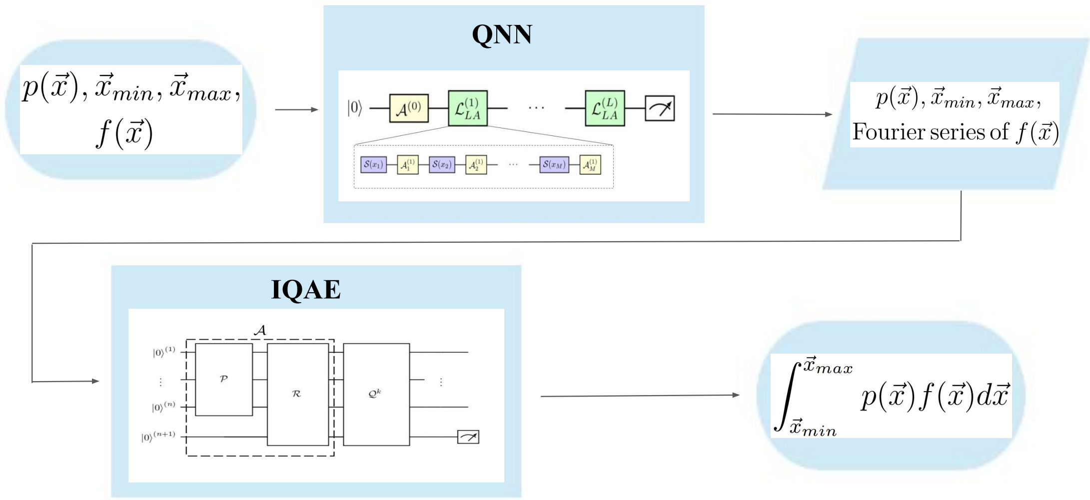

Figure 1: Workflow of QFIAE, figure taken from [20]. First, the function $f(\vec{x})$, the probability distribution $p(\vec{x})$, and the integration interval $\{\vec{x}_{min}, \vec{x}_{max}\}$ are introduced as inputs. Second, the QNN fits $f(\vec{x})$ and the Fourier series is extracted from the quantum circuit. Next, IQAE estimates the integral for each trigonometric piece in the Fourier series. Last, the integrals are added with their corresponding coefficients to obtain the final integral result.

we compute the decay rates of different processes that occur at the CERN's Large Hadron Collider (LHC) as a proof of concept for how quantum computing can provide complete theoretical predictions in the field of particle physics. In particular, we consider at NLO the decay of a Higgs boson or a photon into a quark-antiquark pair, and the decay of a heavy scalar into lighter scalar particles. The integrals these decay processes involved correspond to one-loop Feynman integrals and the phase-space integration of tree-level Feynman diagrams. These integrals are two-dimensional functions.

As depicted in Fig. 1, the first step of QFIAE is to Fourier decompose the target function, i.e. the integrands using a QNN. Fig. 2 shows the Ansatz we consider to train the QNN to approximate the integrands in the domain of integration. Once, we have trained the QNN, we can obtain the Fourier

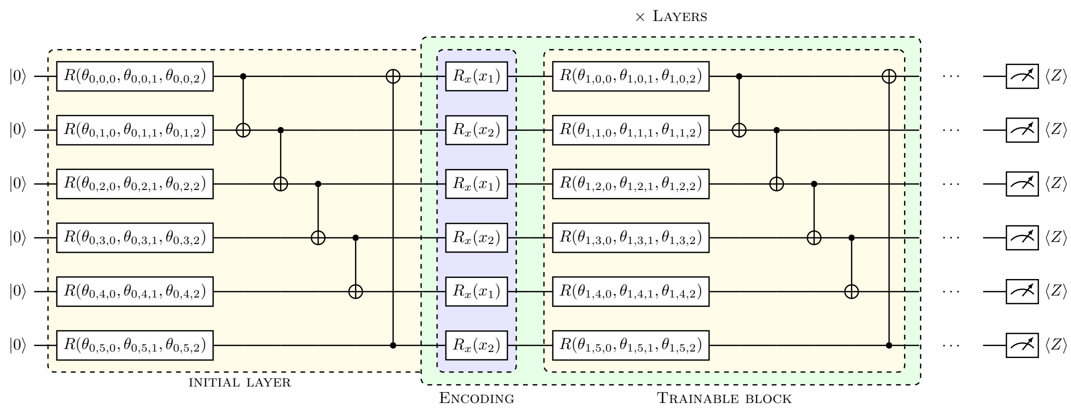

Figure 2: Architecture of the QNN employed to fit a 2-dimensional function.

series that the quantum circuit represents [16] and subsequently apply IQAE to each trigonometric term to finally derive the result of the decay rate. This has been done for the aforementioned processes and the results obtained are displayed in Fig. 3, where the training of the QNN has been done in a quantum simulator using Pennylane [25], whereas the integration has been performed in Qibo simulators [26] (left) and IBMQ hardware using Qiskit [27] (right).

The results of the decay rate integrals obtained in Fig. 3 show a good agreement with the analytical value obtained using state-of-the-art Dimensional Regularization (DREG).

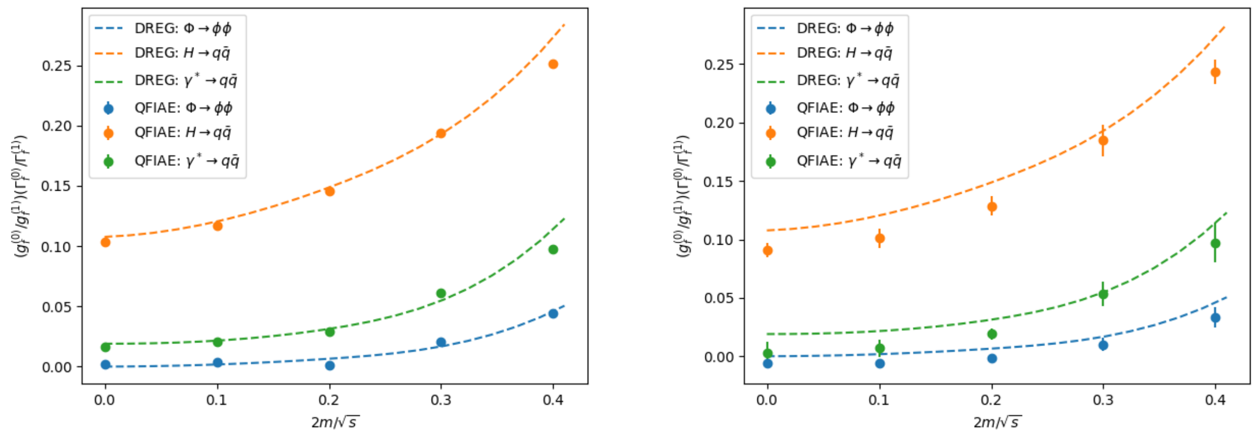

Figure 3: Quantum integration for the three decay processes $H \to q\bar{q}$, $\gamma^* \to q\bar{q}$ and $\Phi \to \phi\phi$ at NLO as a function of the final state mass. The parameters used in the quantum implementation are: $max\_steps = 15000$, $step\_size = 0.001$, $layers = n_{Fourier} = 20$, $n_{qubits} = 6$ for the QNN and $n_{qubits} = 5$, $n_{shots} = 10^3$, $\epsilon = 0.01$, and $\alpha = 0.05$ for the IQAE part. The IQAE component has been performed in a quantum simulator by Qibo (left) and on the 27-qubit IBMQ superconducting device *ibmq_mumbai* (right).

Hence, these results set a precedent as the first application in quantum hardware of an end-to-end quantum algorithm for integrating a physical observable of a particle physics process beyond the leading perturbative order. Not only does this constitute a considerable novelty in the field of particle physics but also in the broader field of quantum computing and quantum machine learning, as it involves solving a regression problem for a highly non-trivial two-dimensional function using a QNN, which, to the best of our knowledge, does not have an antecedent in the literature.

Despite the proliferation of algorithms and new methods in the field of quantum computing, many of these endeavors have fallen short of addressing real-world challenges. Bridging this gap is essential, as tackling real-world problems offers invaluable insights into the limitations of current algorithms and methods. Engaging with such challenges provides a deeper understanding of existing knowledge gaps, facilitating the development of innovative algorithms and solutions. Thus, we believe our work represents a significant achievement in this direction, as we have successfully tackled with a full quantum pipeline the integration of complex Feynman loop integrals associated with the decay processes of elementary particles.

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