Quantum integration of decay rates in perturbation theory

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1. Summary

- Quantum computing to **speed-up** Monte Carlo integration
- New algorithm: QFIAE[1] $\begin{cases} \text{Quantum Neural Network} \longrightarrow \textbf{Fourier series} \\ \text{Monte Carlo } \textbf{integration} \longrightarrow \textbf{IQAE} \end{cases}$
- Proven achievements in physical processes [2]:
  - Multidimensional integration of **Higgs decay** and other processes in quantum hardware.
- First step to **accelerate** Quantum Field Theory **predictions** in **High-Energy Physics (HEP)**

2. Quantum Monte Carlo Integration

Monte Carlo Integration aims to calculate:

$$I = \int_{x_{min}}^{x_{max}} p(x) f(x) dx, \xrightarrow{\text{discretizing}} \mathbb{E}[f(x)] = \sum_{x=0}^{2^n-1} p(x) f(x).$$

A quantum algorithm could estimate this sum with a **quadratic speedup**:

QUANTUM AMPLITUDE ESTIMATION

We encode $p(x)$ and $f(x)$ into quantum circuits:

$$\mathcal{P}|0\rangle_n = \sum_{x=0}^{2^n-1} \sqrt{p(x)} |x\rangle_n, \quad \mathcal{R}|x\rangle_n|0\rangle = |x\rangle_n \left(\sqrt{f(x)}|1\rangle + \sqrt{1-f(x)}|0\rangle \right),$$

Combining both operators one builds the operator $\mathcal{A} = \mathcal{R}(\mathcal{P} \otimes \mathbb{I}^1)$:

$$|\psi\rangle = \mathcal{A}|0\rangle_{n+1} = \sum_{x=0}^{2^n-1} \sqrt{p(x)} |x\rangle_n \left(\sqrt{f(x)}|1\rangle + \sqrt{1-f(x)}|0\rangle \right)$$

$$= \sqrt{a}|\tilde{\psi}_1\rangle|1\rangle + \sqrt{1-a}|\tilde{\psi}_0\rangle|0\rangle, \quad \text{with} \quad a = \mathbb{E}[f(x)].$$

Now, with the amplification operator $\mathcal{Q}$, we apply Grover's algorithm:

$$\mathcal{Q}^k \mathcal{A}|0\rangle_{n+1} = \sin((2k+1)\theta_a)|\tilde{\psi}_1\rangle|1\rangle + \cos((2k+1)\theta_a)|\tilde{\psi}_0\rangle|0\rangle,$$

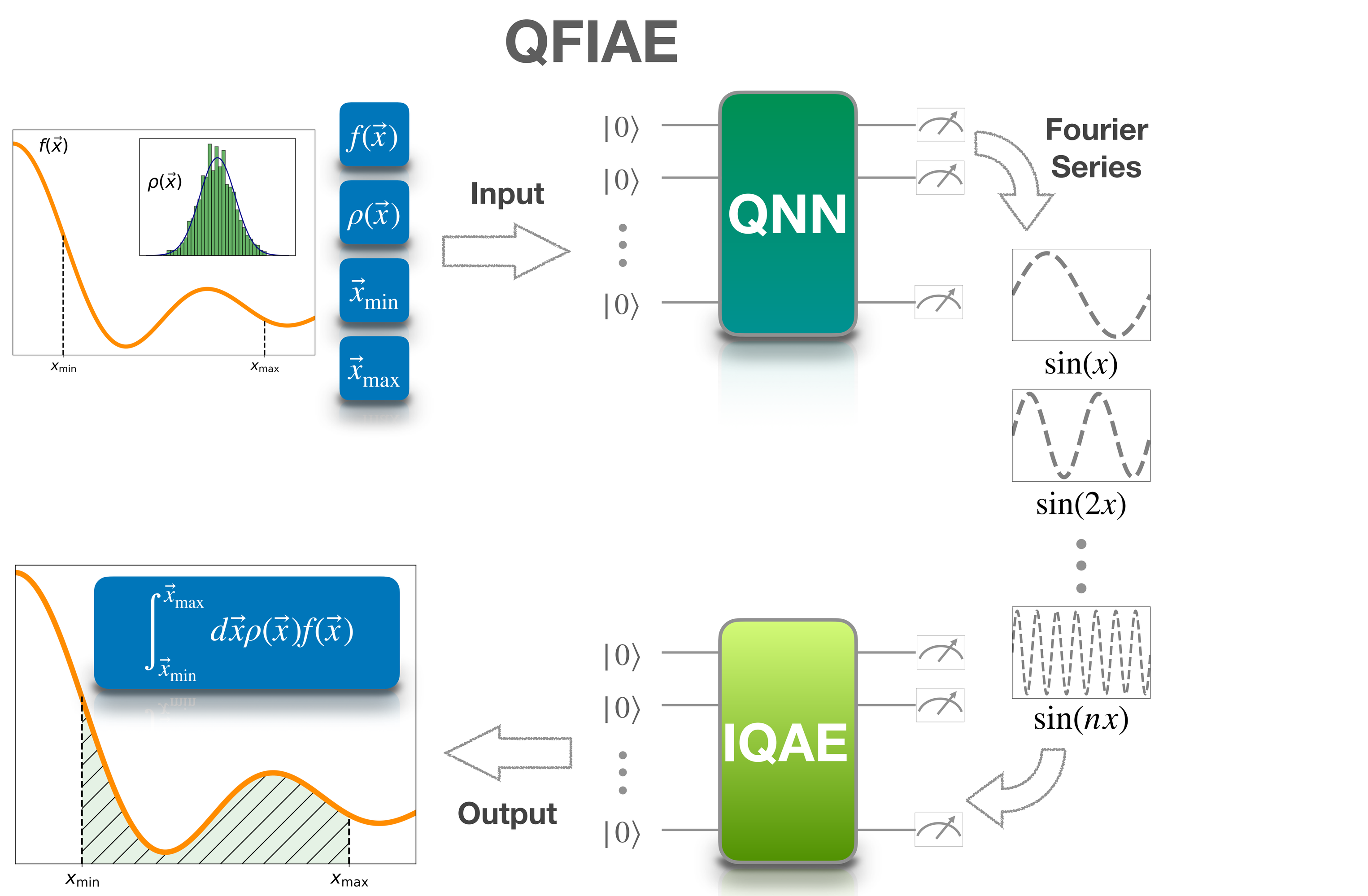
$$\text{where} \quad a = \sin^2\theta_a, \quad \mathcal{Q} = -\mathcal{A}S_0\mathcal{A}^{-1}S_\chi,$$

$$S_\chi|\tilde{\psi}_1\rangle|1\rangle = -|\tilde{\psi}_1\rangle|1\rangle, \quad S_0|0_{n+1}\rangle = -|0_{n+1}\rangle$$

Finally, we infer the ratio of the **good state** $|\tilde{\psi}_1\rangle|1\rangle$ over the **bad state** $|\tilde{\psi}_0\rangle|0\rangle$ and estimate the value of a from such a ratio.

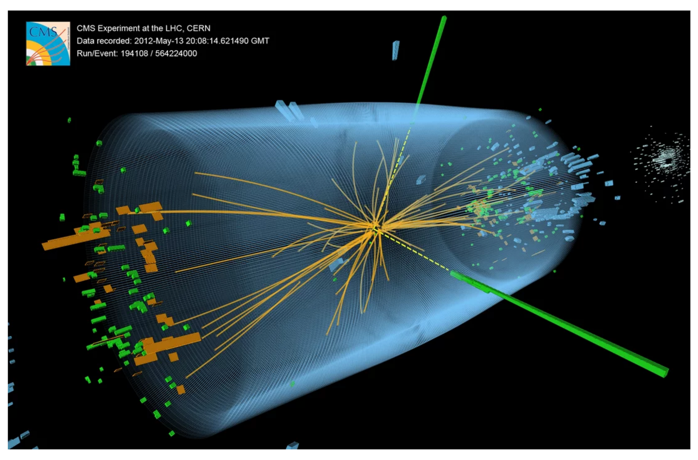
3. Quantum Fourier Iterative Amplitude Estimation

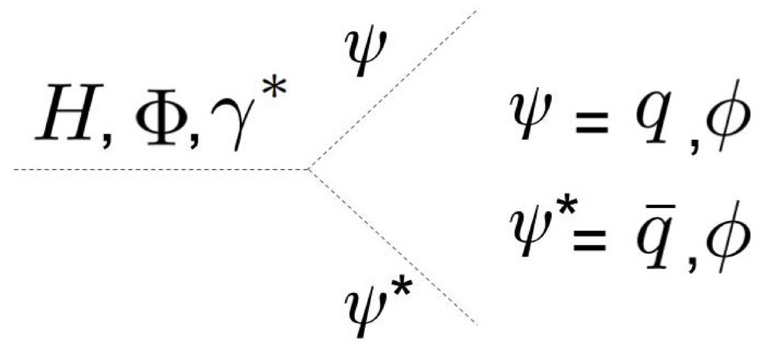
- Main **challenge** of QMCI: **encode** any $f(\vec{x})$ function into $\mathcal{R}$
- Proposed **solution**: QFIAE [1]
  - Decompose $f(\vec{x})$ into a Fourier series [2] through a Quantum Neural Network [3]
  - Apply Iterative QAE (IQAE)[4] to every Fourier series term



4. Decays and integrals

We consider the following particle decays at particle physics colliders:



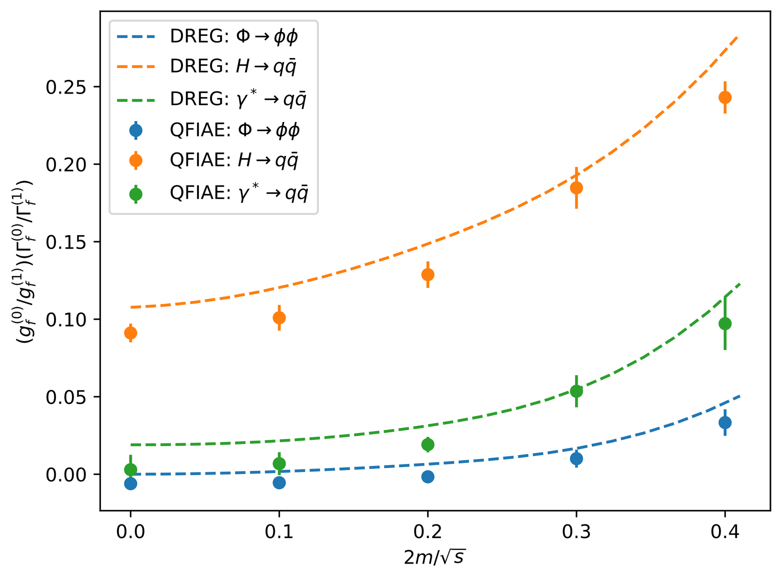


5. Quantum implementation

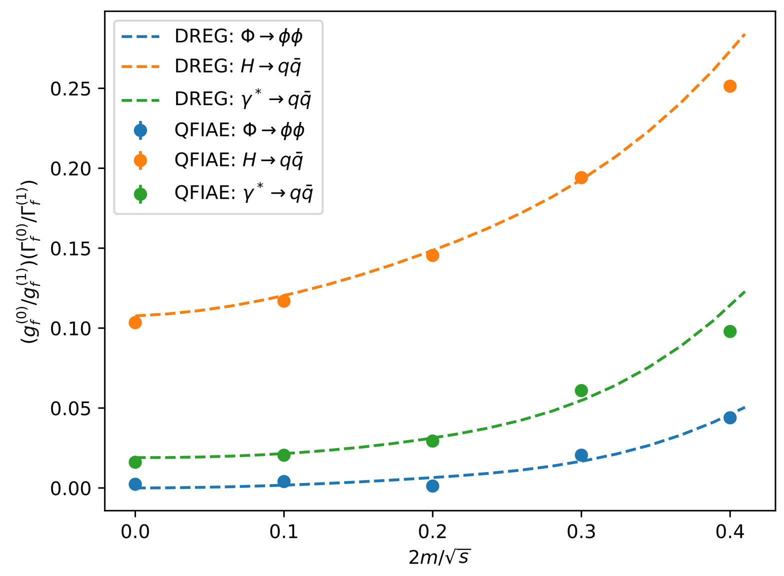
We compute the decay rates integrating quantumly the mentioned Feynman loop integrals and compare with classical values:

INTEGRATION ON QUANTUM

HARDWARE

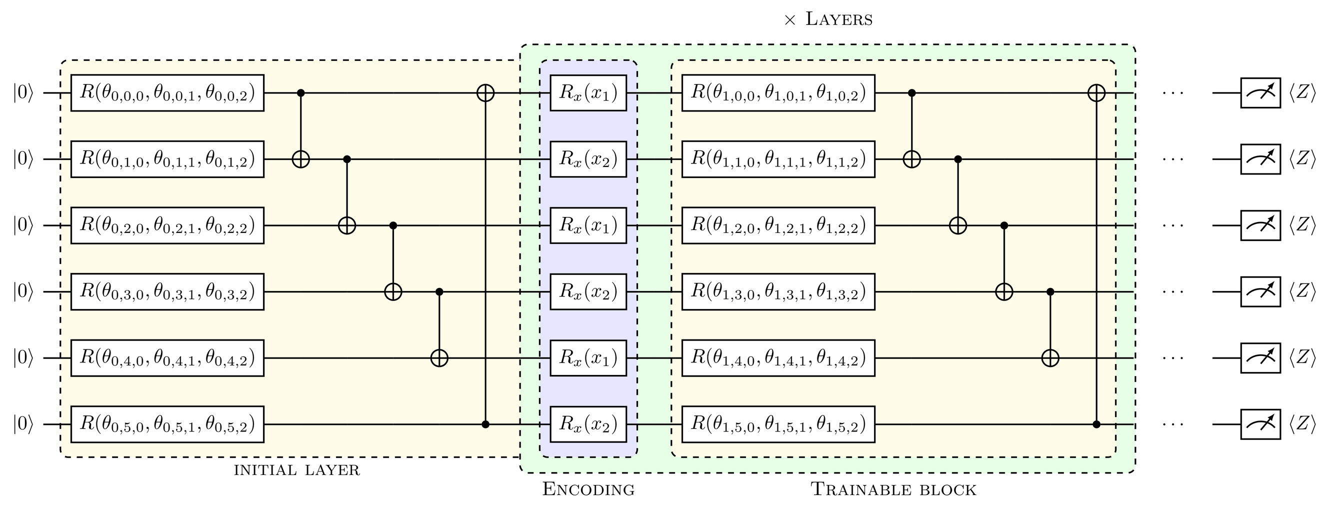


SIMULATOR



Decay rates integration on simulator `Pennylane` (for QNN) and IBMQ hardware/simulator (for IQAE).

ARCHITECTURE OF QNN



6. Conclusions

1. **Decay rates** are **essential** for understanding particle behavior at **high-energy** colliders, quantifying how **quickly** a particle decays.
2. We explore the case where **quantum computing** could **speed up** the calculation of **decay rates integrals**.
3. We consider a new quantum algorithm, **Quantum Fourier Iterative Amplitude Estimation** (QFIAE) that enables Monte Carlo integration of **multidimensional** functions with a theoretical quadratic **speedup**.
4. We have **successfully** implemented the integration part of QFIAE on **hardware**.

7. References

[1] J.J.M. de Lejarza, M. Grossi, L. Cieri and G. Rodrigo, *Quantum Fourier Iterative Amplitude Estimation*, *IEEE International Conference on Quantum Computing and Engineering (QCE) (2023)*, **Vol. 1 pp. 571–579**.

[2] J.J.M. de Lejarza, David F. Rentería-Estrada, M. Grossi and G. Rodrigo, *Quantum integration of decay rates at second order in perturbation theory*, arXiv:2409.12236 (2024).

[3] S. Herbert, *Quantum monte carlo integration: The full advantage in minimal circuit depth*, *Quantum* **6** (2022) 823.

[4] M. Schuld, R. Sweke and J.J. Meyer, *Effect of data encoding on the expressive power of variational quantum-machine-learning models*, *Physical Review A* **103** (2021) 032430.

[5] D. Grinko, J. Gacon, C. Zoufal and S. Woerner, *Iterative quantum amplitude estimation*, *npj Quantum Information* **7** (2021).