Learning multi-time quantum processes

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Quantum learning theory is an exciting field at the intersection of quantum information and machine learning, where researchers seek to establish the difficulty in reconstructing distributions that arise in quantum mechanics. As one example, a recent line of thinking has been to consider the task of predicting outcomes of observables over a range of inputs under some dynamical evolution. In Ref. [1], it is surprisingly shown that this can be made efficient regardless of the complexity of the process. Specifically, for (possibly highly entangled) inputs sampled from a locally flat distribution, there exists a sample- and time-efficient machine learning algorithm capable of accurately predicting the expectation value of bounded degree observables, on average. Here, we continue this research direction and consider the application of machine learning algorithms for predicting properties of multi-time quantum processes.

Multi-time processes describe sequential quantum channels with memory, also known as non-Markovian quantum dynamics. Understanding the behaviour of non-Markovian processes has applications across quantum chemistry, quantum computing, and quantum machine learning. Examples include energy transfer in photosynthesis [2], characterising and accounting for noise in quantum devices [3], and using quantum memory for advantage in learning tasks [4]. The general set-up of a multi-time process that we consider is that of an accessible system coupled to an inaccessible environment, where the composite system-environment evolves according to unitary dynamics. An experimenter may probe the accessible system at various times with a series of control operations in order to learn information about the underlying system-environment interaction. These operations are general quantum instruments representing all possible manipulations of the system, such as a measurement and feed-forward or unitary rotations, and are mathematically described by completely positive maps. Strong system-environment correlations may mediate an exchange of information between the two, meaning that the choice of operation at one time can affect the future dynamics – generating complex multi-time statistics. At the end of a k-step process one therefore obtains a series of outcomes conditioned on the experimenter chosen sequence of k interventions. This quantum control scheme hence constitutes a natural dynamic sampling task: the outcomes describe a probability distribution in time. This motivates a learning task for multi-time processes: given access to a series of measurements and preparations obtained by sampling a k-step process, what can we efficiently learn about the outcome of a sequence of operations – that is, what can we learn about the underlying probability distribution?

We apply the analysis of Ref. [1] to multi-time processes with varying degrees of memory. On the one hand, we show that learning the outcome of any bounded degree multi-time observable on a Markovian (or memoryless) process requires a sample complexity that is quasi-polynomial in the number of sampling times k. This means that one can efficiently learn the outcome of sampling with unitary operations at each time of a Markovian process. On the other hand, processes with an arbitrary degree of memory require $\mathcal{O}(d^k)$ samples, for a system with dimension d. Whilst this is an improvement on the $\mathcal{O}(d^{4k})$ samples required for full tomography of a multi-time process [5], this still presents an exponential scaling in complexity. However, surprisingly, a subset of these processes remain efficiently learnable. We show that non-Markovian processes with exponentially small singular values can be efficiently learned for the purposes of predicting bounded degree observables. Moreover, if we consider operations that constitute a measurement at each time followed by the preparation and feed-forward of a Haar random state, then we can efficiently learn the series of outcomes on any arbitrary multi-time process to ϵ average-case error. This measurement scenario is sufficient to learn all multi-time correlations under the defined setting. We are exploring the existence of alternative algorithms that may enable efficient learning of different classes of multi-time processes and observables.

Our results highlight the potential for machine learning algorithms to learn complex quantum temporal correlations. Applications include quantum control optimisation for non-Markovian dynamics, and optimising training of variational quantum algorithms. Furthermore, the methods used provide a framework for studying the complexity separation between quantum processes and quantum states, and may help to formally determine if properties, such as quantum memory, are fundamental indicators of complexity.

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