Variational State Preparation with Noisy Trapped-Ion Quantum Computers

D. Rabinovich[1], Z. Sayapin[1,*], E. Campos[1], S. Adhikary[2]

[*]Z.Sayapin@skoltech.ru

[1]Skolkovo Institute of Science and Technology, Moscow, Russia

[2]Centre for quantum technologies, National University of Singapore, Singapore

Abstract

Noise in quantum devices challenges implementation of quantum algorithms. Our study focuses on noisy trapped-ion quantum computers, addressing the errors arising from residual entanglement between electronic and motional levels. We introduce a variational quantum state preparation algorithm for non-ideal ionic quantum computers, achieving in our numerical experiments fidelities in preparing GHZ states with 3-5 qubits up to 0.99. Additionally, we propose a solution to optimize the hardware requirements for mixed-state preparation on ion quantum computers, using platform-specific motional modes as a useful computational resource.

Noisy trapped-ion quantum computers

The trapped ions platform is currently one of the leading platforms for quantum computing owing to their long coherence times, all-to-all connectivity, and availability of the required technologies.

Each ion in such setups is treated as a qubit, with the logical levels of the qubit encoded in the electronic states of the ion. Using suitable laser pulses, a universal set of gates can be performed. In particular, we use a set of gates consisting of ideal single-qubit rotations R_x and R_z given in (1) and (2) respectively and realistic entangling Molmer-Sorensen (MS) gate provided in equations (3), (4) and (5).

$$R_x(t) = \exp\left(-i\frac{\Omega t}{2}\sigma^x\right), \tag{1}$$

$$R_z(t) = \exp\left(-i\frac{t\Omega^2}{4\Delta}\sigma^z\right), \tag{2}$$

Here, Ω is Rabi frequency, $\Delta = \omega - \omega_0$ is the detuning of the laser frequency ω from the frequency of transition ω_0 between states $|0\rangle$ and $|1\rangle$; σ^x and σ^z are the Pauli operators.

$$U_{MS}^{real} = \exp\left\{ i\sum_{p,q}\Phi_{p,q}\sigma_p^x\sigma_q^x + \sum_{p,m}[\alpha_p^m a_m^\dagger - \alpha_p^{m*}a_m]\sigma_p^x \right\}, \tag{3}$$

where

$$\alpha_p^m(t) = i\eta_p^m \int_{t_0}^{t} \Omega_p(t')\sin(\Delta t')e^{i\nu_m t'}dt' \tag{4}$$

$$\Phi_{p,q}(t) = \sum_m \eta_p^m\eta_q^m \int_{t_0}^{t}\int_{t_0}^{t'} \Omega_p(t')\Omega_q(t'')\sin(\Delta t')\sin(\Delta t'')\sin(\nu_m(t'-t''))dt''dt' \tag{5}$$

Here, t_0 is the initial time, Ω_p is Rabi frequency for the p-th ion, σ_p^x is the Pauli-X operator acting on the p-th qubit, $a_m^\dagger$ and a_m are the creation and annihilation operators respectively of mode m, ν_m is a frequency of m-th motional mode, η_p^m is the Lamb-Dicke parameter associated with m-th motional mode and ion p-th ion.

Variational state preparation

Given a target n-qubit pure state $|t\rangle$ and an ansatz $V(\boldsymbol{\theta})$ we find a set of parameters $\boldsymbol{\theta}_\star$ such that $|\psi(\boldsymbol{\theta}_\star)\rangle = V(\boldsymbol{\theta}_\star)|0^n\rangle$ approximates $|t\rangle$. The cost function here is simply given by expectation value of the following Hamiltonian (6).

$$H = (1 - |t\rangle\langle t|) \otimes \mathbb{1} \tag{6}$$

where $\mathbb{1}$ is an operator that acts trivially on the infinite dimensional Hilbert space corresponding to the motional degrees of freedom $(\mathcal{H}^\infty)^{\otimes M}$ and M is the number of motional modes considered. Clearly the states minimizing H are of the form $|t\rangle \otimes |\varphi\rangle$, where $|\varphi\rangle \in (\mathcal{H}^\infty)^{\otimes M}$. Taking a partial trace of this state over $(\mathcal{H}^\infty)^{\otimes M}$ gives us our desired target state.

For mixed state preparation we minimize the Hilbert-Schmidt distance $\mathrm{HS}(\rho_t, \rho_p)$ defined in (7) between target mixed state ρ_t and a state ρ_p prepared in a register.

$$\mathrm{HS}(\rho_t, \rho_p) = \sqrt{\mathrm{Tr}(\rho_t - \rho_p)^2} \tag{7}$$

Ansatz

The variational circuit $V(\boldsymbol{\theta})$ is composed of p layers of checkerboard ansatz shown in Fig.1. For each block in the ansatz, we have 7 variational parameters consisting of 4 angles for each single-qubit rotation and 3 laser pulse parameters for the MS gate: the duration of the pulse $t - t_0$, the detunig of the laser Δ and the Rabi frequency Ω. We assume that the Rabi frequency is constant during the laser pulse.

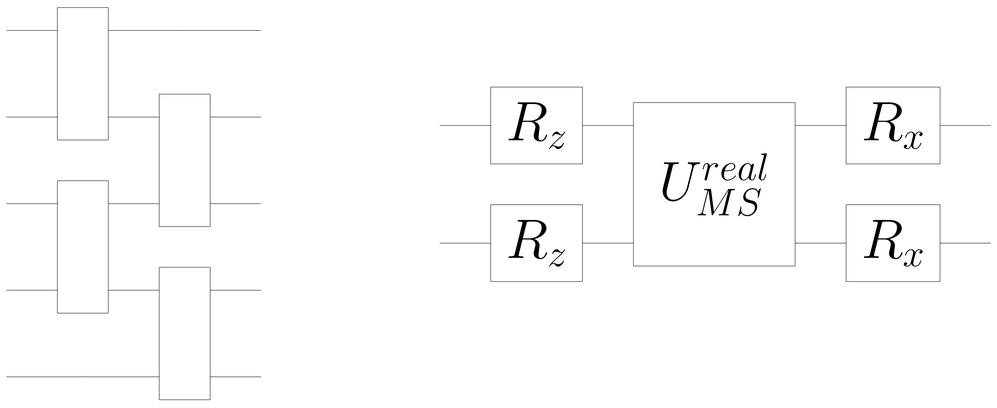

Fig. 1: Left: The checkerboard ansatz for $n = 5$. It is built of identical blocks arranged in a checkerboard pattern. Right: Structure of each block in the checkerboard ansatz.

Pure state preparation

For the sake of numerical simulation we approximate the infinite dimensional Hilbert space that $V(\boldsymbol{\theta})$ acts on (i.e. $(\mathbb{C}_2)^{\otimes n} \times (\mathcal{H}^\infty)^{\otimes M}$), by a finite dimensional Hilbert space $(\mathbb{C}_2)^{\otimes n} \times (\mathcal{H}^N)^{\otimes M}$. Then we obtain the minimum number of layers required for successful state preparation a.k.a. the critical depth p^*. We deem our variational state preparation algorithm to be successful if we achieve $1 - |\langle t|\psi_p(\boldsymbol{\theta})\rangle|^2 \leq 10^{-1}$. The Tab. 1 shows the results for GHZ state preparation considering single $M = 1$ center-of-mass (COM) mode.

n	p^*	N_{min}	$1 - \lvert\langle t\rvert\psi(\boldsymbol{\theta})\rangle\rvert^2$
3	3	16	0.07
4	3	32	0.03
5	4	32	0.01

Tab. 1: Summary of GHZ state preparation for $n = 3, 4, 5$ qubits and COM mode. N_{min} is the minimum number of motional levels that were required in numerical experiments.

We can further improve the performance of our algorithm with introduction of additional layers. In fact we can achieve $1 - |\langle t|\psi_p(\boldsymbol{\theta})\rangle|^2 \leq 10^{-2}$ with $p = 4, 4, 6$ for $n = 3, 4, 5$ respectively.

Mixed state preparation

We demonstrate that it is possible to prepare mixed states in a register without the use of ancilla qubits, exploiting the entanglement between the electronic and motional subsystems. In numerical experiments, mixed states were prepared for 2 qubits that differed from the target mixed states in terms of the Hilbert-Schmidt distance by no more than 0.01. The example of a prepared state is shown in Fig. 2.

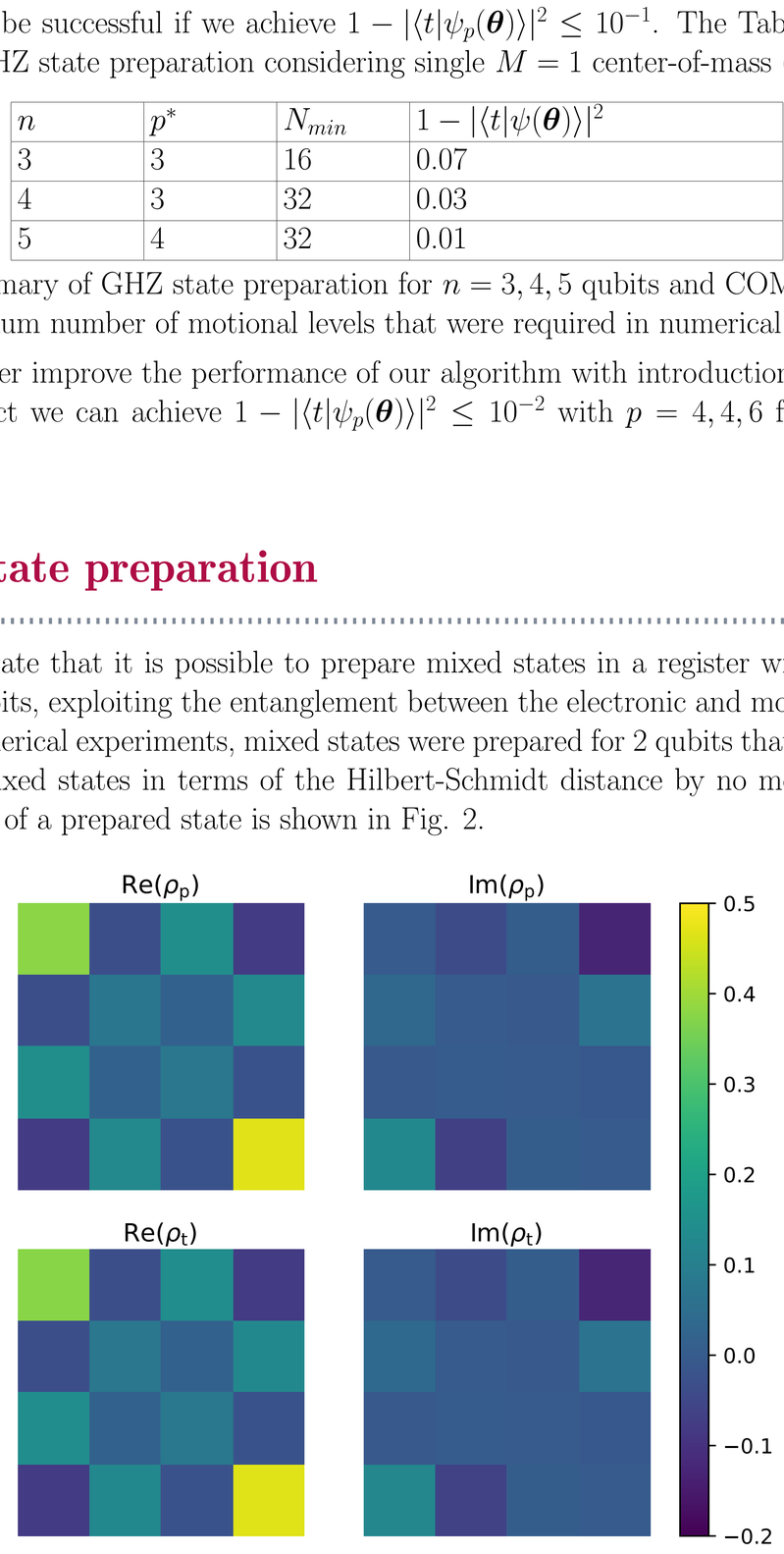

Fig. 2: Real and imaginary components of target ρ_t and prepared ρ_p density matrices on $n = 2$ qubits for single $M = 1$ COM mode approximated with $N = 16$ motional levels, $p = 6$, the optimized Hilbert-Schmidt distance $\mathrm{HS}(\rho_t, \rho_p) = 0.009$

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