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Mind the QApp - Visualizing the Crux in QML

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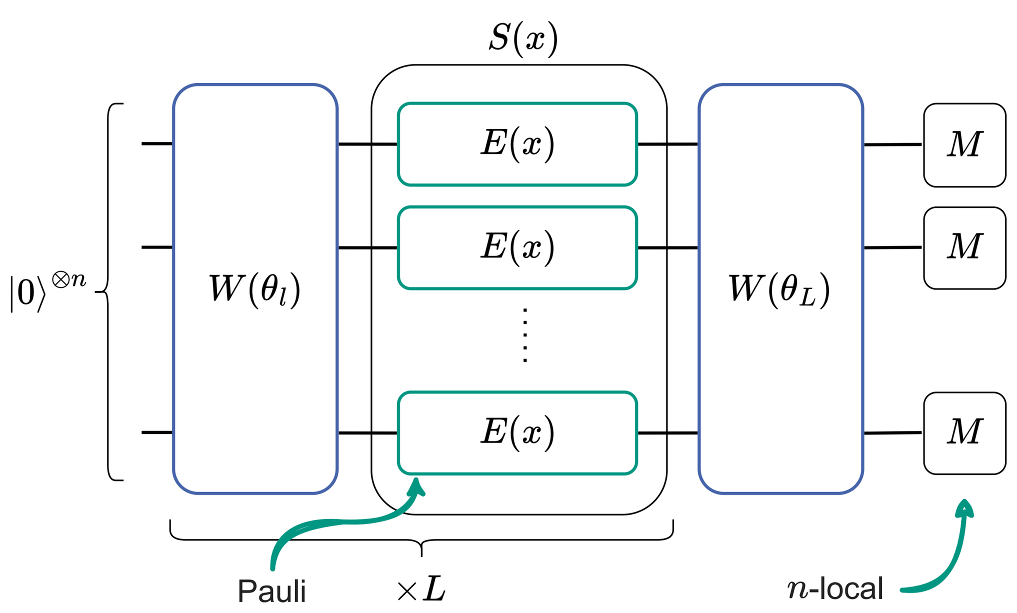
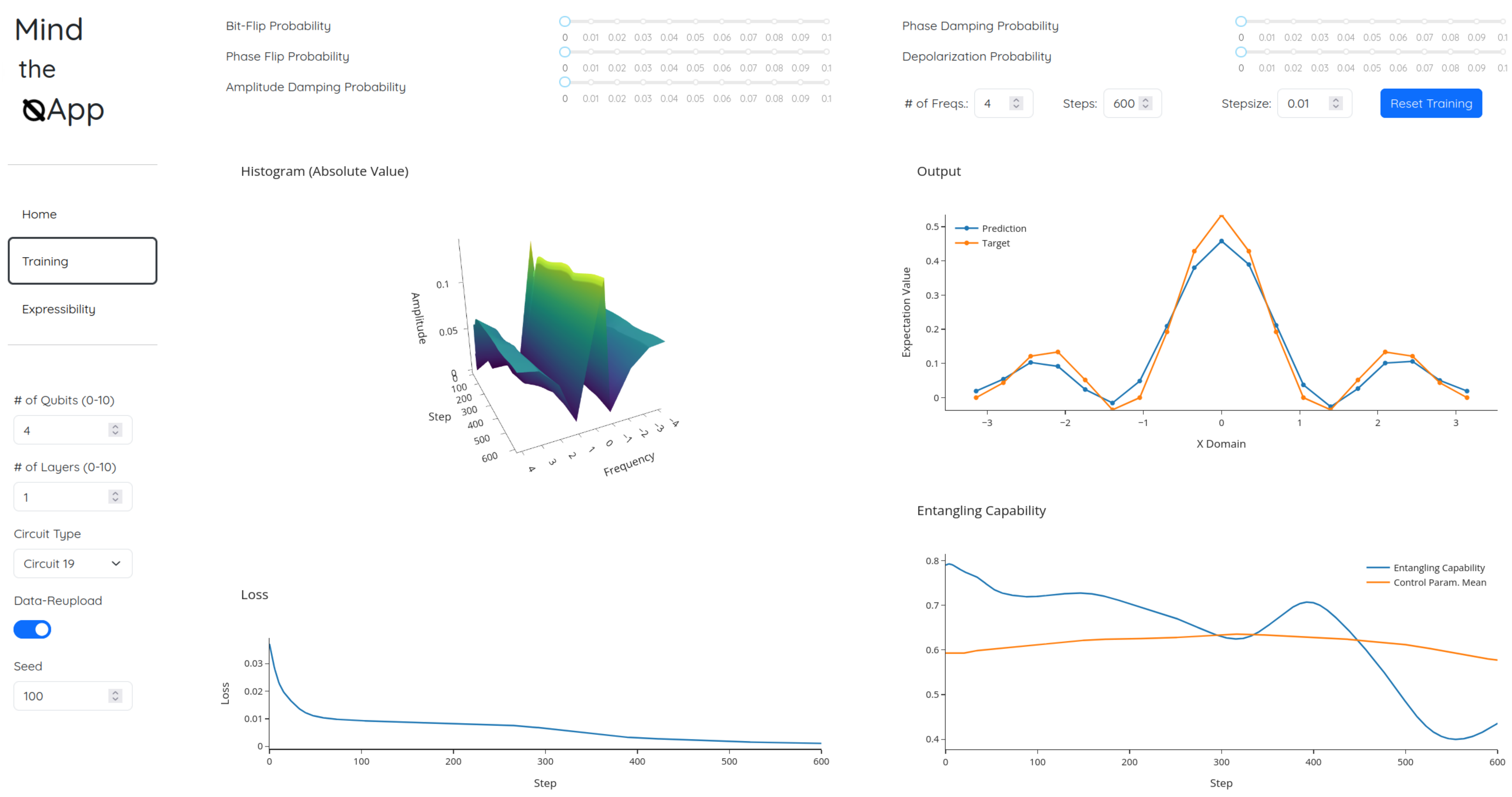
Overview

- Browser-App (Python+Dash) to visualize basic principles in QML:
  - Data-Reuploading Model [1]
  - Training on Fourier series dataset
  - Expressibility & Entangling Capability
- Ansatz specific differences in learning behavior/ effect of
  - Entangling Capability
  - Expressibility
- Assessing the impact of noise





Fourier Models

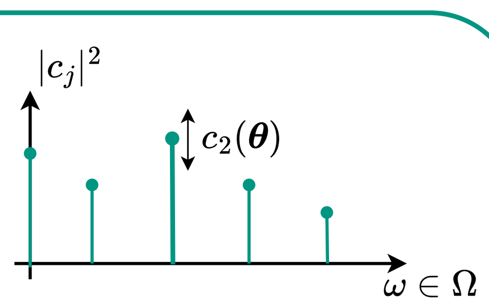
Equivalence between Data-Reuploading Model and Fourier series as shown in Schuld et al. [1]

$$U(x,\theta) = W^{(L+1)}S(x)W^{(L)}\dots W^{(2)}S(x)W^{(1)}$$

$$f(x,\boldsymbol{\theta}) = \langle \mathbf{0} \left| U^\dagger(x,\boldsymbol{\theta}) M U(x,\boldsymbol{\theta}) \right| \mathbf{0} \rangle$$

[1] $\vdots$

$$= \sum_{\omega\in\Omega} c_\omega e^{i\omega x}$$



$$\Omega = \left\{\Lambda_{\boldsymbol{k}} - \Lambda_{\boldsymbol{j}}, \boldsymbol{k}, \boldsymbol{j} \in [d]^{(R*L)}\right\}$$

$d = 2$ for Pauli Encoding

$$\Lambda_j = \lambda_j^1 + \dots + \lambda_j^{(R*L)}$$

Entanglement

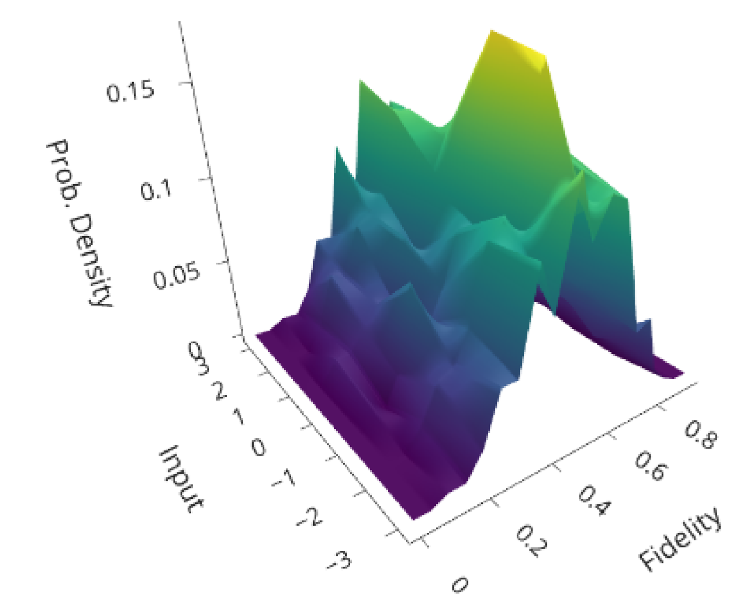
Calculation of entanglement via method proposed in Brennen et al. [3], based on the Meyer-Wallach measure [2].

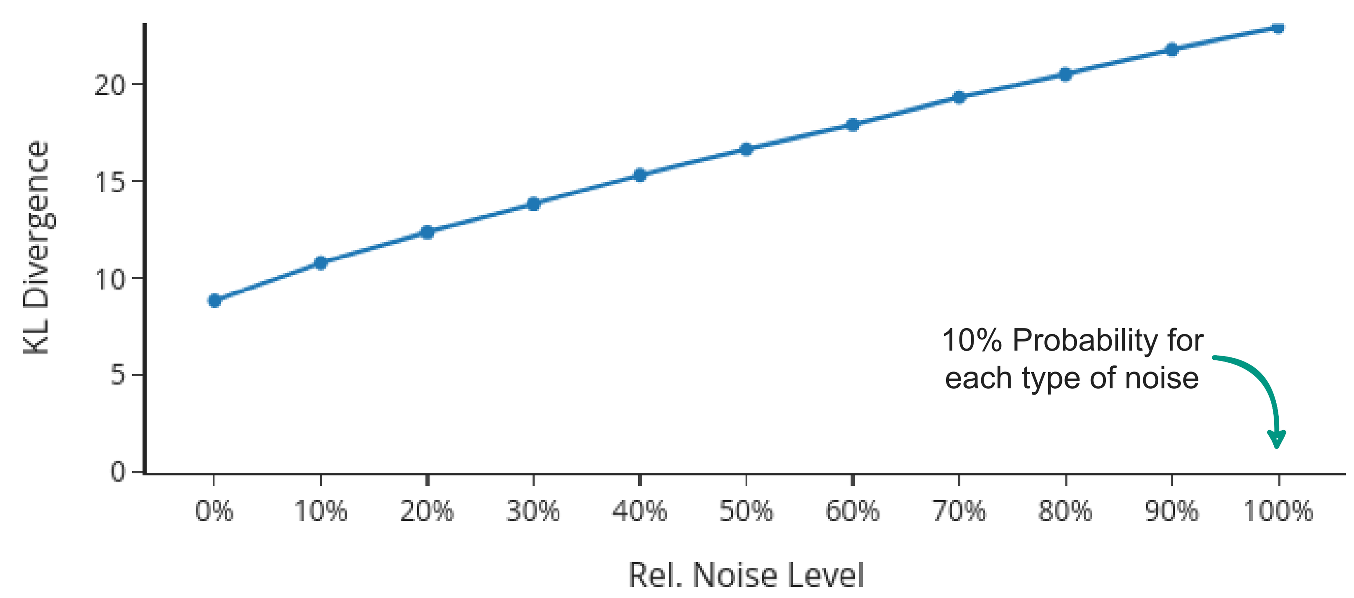
$$Q(|\psi\rangle) = 2\left(1 - 1/n \sum_{k=0}^{n-1} \mathrm{Tr}\left[\rho_k^2\right]\right)$$

with ρ_k being the traced-out density matrix of qubit k

- When $\mathrm{Tr}[\rho_j^2] = 1 \;\; \forall j \implies Q = 0$ meaning $|\psi\rangle$ is a product state
- $Q = 1 \iff \mathrm{Tr}\left[\rho_k^2\right] = 1/2 \;\; \forall k$ meaning $|\psi\rangle$ is maximally mixed

Expressibility





- Expressibility: Kullback-Leibler (KL) divergence between Haar random states and sampled VQC states [4]
- *Left*: sampled over input values

- *Right*: sampled over different noise levels
- Amplitude/ Phase Damping, Bit/ Phase-Flip, Depolarization

		Fidelity - Probability of State Overlaps	Distribution of State Overlaps (Histogram)
Sampled VQC States	$\int_\Theta (\lvert\psi_\theta\rangle\langle\psi_\theta\rvert)^{\otimes t} d\boldsymbol{\theta}$ →	$F = \lvert\langle\psi_\varphi \mid \psi_\phi\rangle\rvert$ →	$p\left(F = \lvert\langle\psi_\varphi \mid \psi_\phi\rangle\rvert\right)$
Haar Random States	$\int_{\mathrm{Haar}} (\lvert\psi\rangle\langle\psi\rvert)^{\otimes t} d\psi$		

$$\mathrm{Expr} := D_{\mathrm{KL}}\left(\hat{P}_{\mathrm{VQC}}(F;\boldsymbol{\theta}) \| P_{\mathrm{Haar}}(F)\right)$$

Outlook

- How is training affected by
  - entangling capability?
  - entangling gate initialisation?
- How is expressiblity affected by
  - different types of noise?
  - input encoding?

- Deploy as online platform
- Extend by
  - Support of different optimizers
  - Visualize of the loss landscape
- Improve speed of calculating expressibility and entangling capability

cirKITers/qml-essentials

Python package with data-reuploading model, commonly used Ansaetze, and tools in quantum machine learning.

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[1]: Schuld et al. - Effect of data encoding on the expressive power of variational quantum-machine-learning models
[2]: Meyer et al. - Global entanglement in multiparticle systems
[3]: Brennen et al. - An observable measure of entanglement for pure states of multi-qubit systems
[4]: Sim et al. - Expressibility and Entangling Capability of Parameterized Quantum Circuits for Hybrid Quantum-Classical Algorithms

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