Noise-Tolerant Learnability of Shallow Quantum Circuits from Statistics and the Cost of Quantum Pseudorandomness

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Joint work with Mina Doosti

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Motivation

Learning with and about near-term quantum devices.

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- Learning with limited quantum capabilities.
- Learning in the presence of noise.

Outline

Quantum Statistical Query Learning

Robustness of statistical learners

Learning shallow quantum circuits

Low-depth lower bounds

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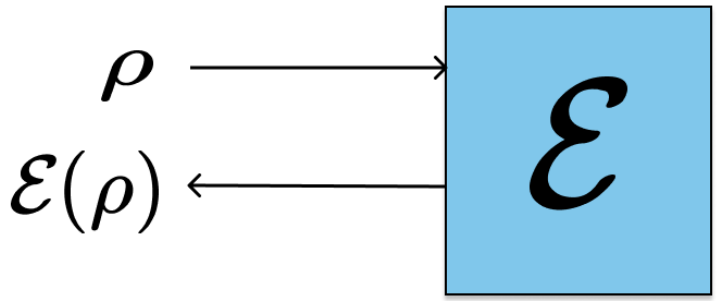
- We will consider the diamond distance.

$$d(\mathcal{E}_1, \mathcal{E}_2) = \frac{\|\mathcal{E}_1 - \mathcal{E}_2\|_\diamond}{2}, \quad \|\mathcal{E}\|_\diamond = \max_\rho \|(\mathcal{E} \otimes \mathcal{I})\rho\|_1.$$

Black-Box Queries

- Conventionally, learning algorithms assume black-box query access to the unknown channel.



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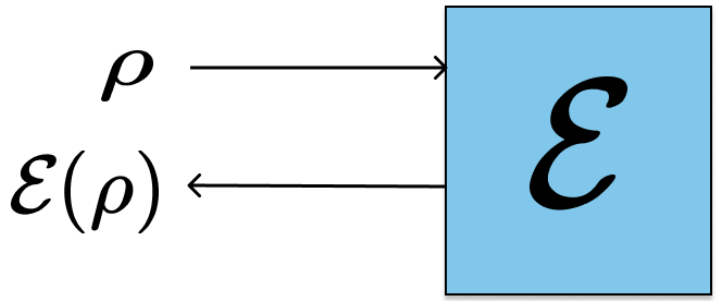
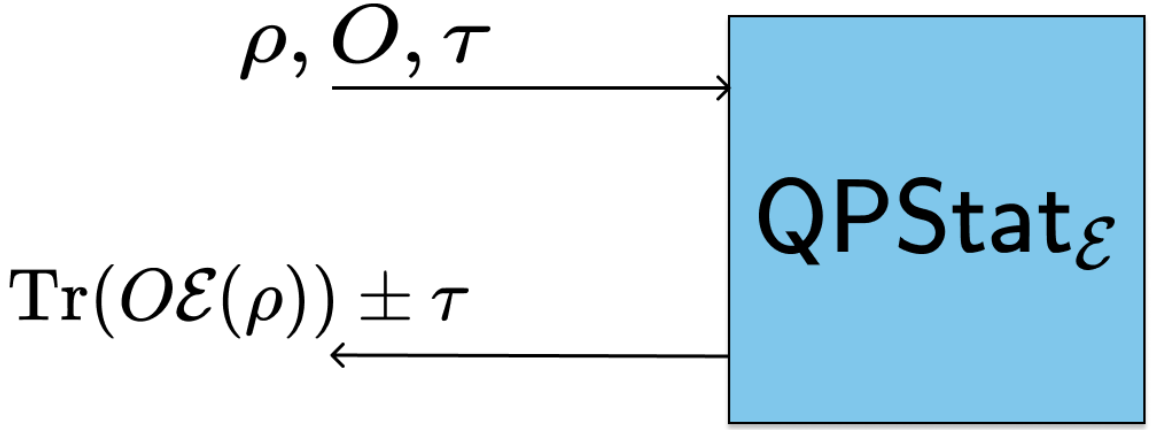


- Demonstrating robustness requires considerable effort.
- Is there another natural access model with robustness guarantees?

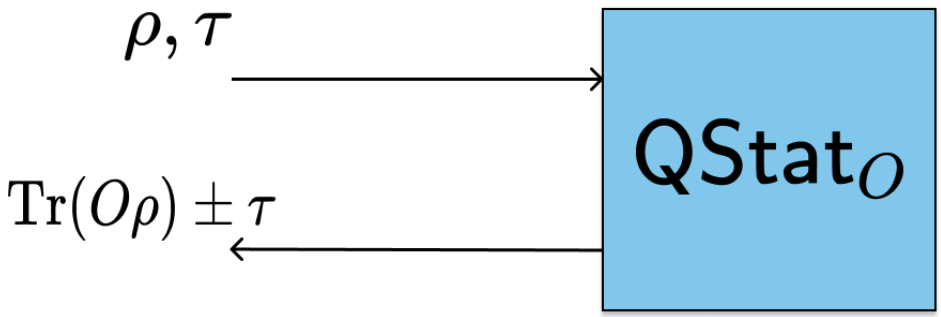
Quantum Statistical Queries to Processes [WD23]

We will assume access to the QPStat oracle.



Other access models

- For the problem of learning an unknown observable, we define the following oracle:



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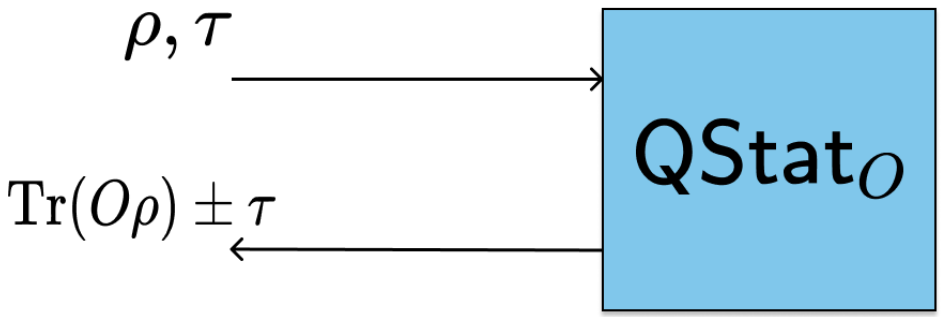

- We also define a "multi-copy" QPSQ oracle.

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Intuition: Noise shifts the output expectation value by at most η.

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- This class can be learned using $\mathcal{O}(n^2 \log(n)/\epsilon^2)$ black-box queries to the unknown circuit [HLB+24].

Learning shallow quantum circuits with QPStat

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Theorem

There exists an algorithm for learning constant-depth circuits of two-qubit gates using $\mathcal{O}(n^3 \log(n)/\epsilon^2)$ QPStat *queries of tolerance* $\Omega(\epsilon/n)$ *and running in time* $\mathrm{poly}(n)/\epsilon^2$.

Proof Idea

Insights from [HLB+24]:

1. In order to learn a unitary U, it suffices to learn the $3n$ Heisenberg-evolved Paulis $U^\dagger P_i U, i \in [n], P \in \{X, Y, Z\}$.
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We show:

- A local observable O can be learned efficiently from queries to QStat_O.
- Using queries to QPStat_U, we can simulate queries to $\mathsf{QStat}_{U^\dagger P_i U}$
- We learn the Heisenberg-evolved Paulis from queries to QPStat, allowing us to learn the circuit!

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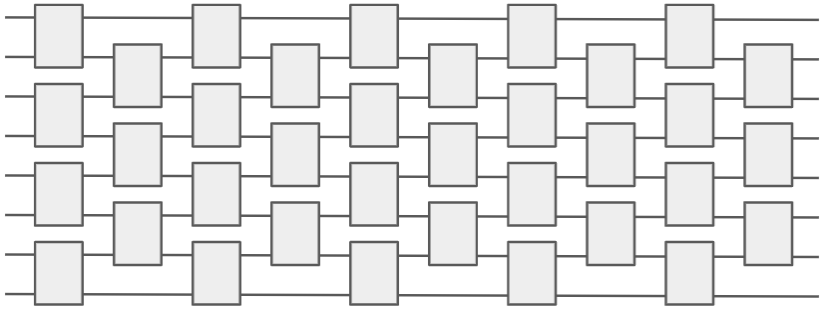
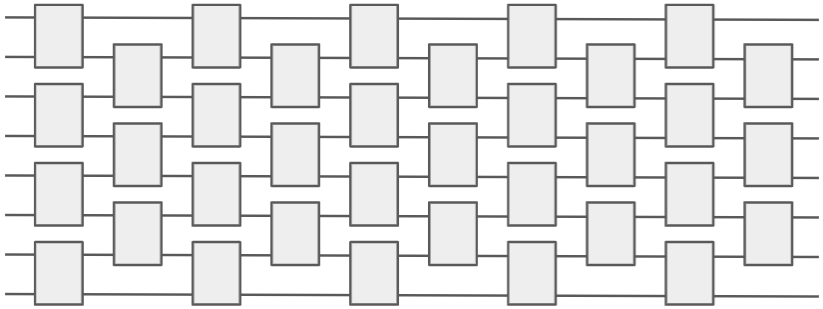
Average-case lower bounds for shallow quantum circuits



Brickwork random quantum circuits (BRQCs)

Average-case lower bounds for shallow quantum circuits

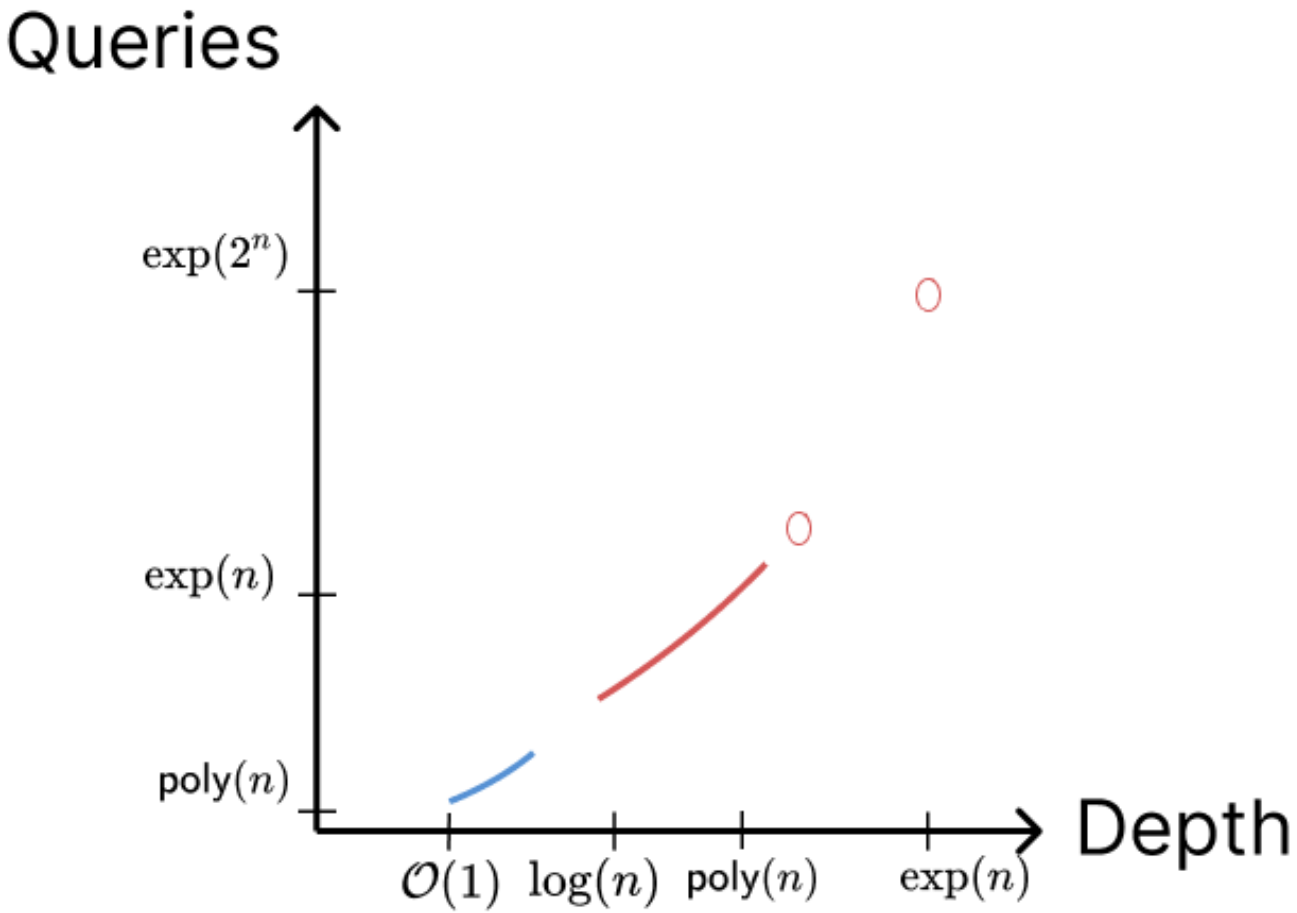


Brickwork random quantum circuits (BRQCs)

Theorem

Any algorithm for learning BRQCs of depth d with high probability using QPStat *queries of tolerance τ must make $\Omega\left(\frac{\tau^2}{n}\left(\frac{5}{4}\right)^d\right)$ queries, when $\log n \leq C \cdot d \leq n + \log n$, for a fixed constant C.*

Query complexity of learning BRQCs



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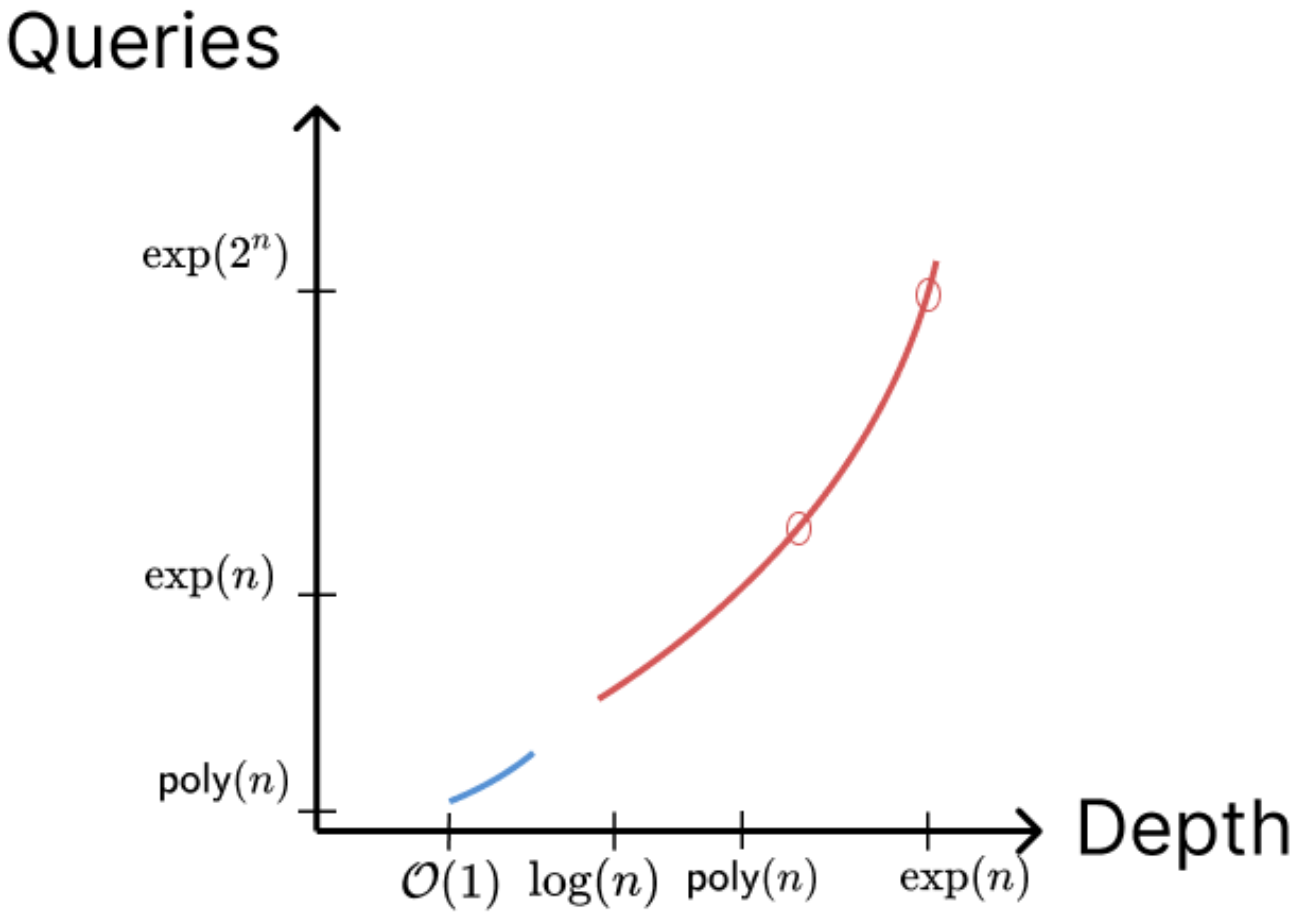

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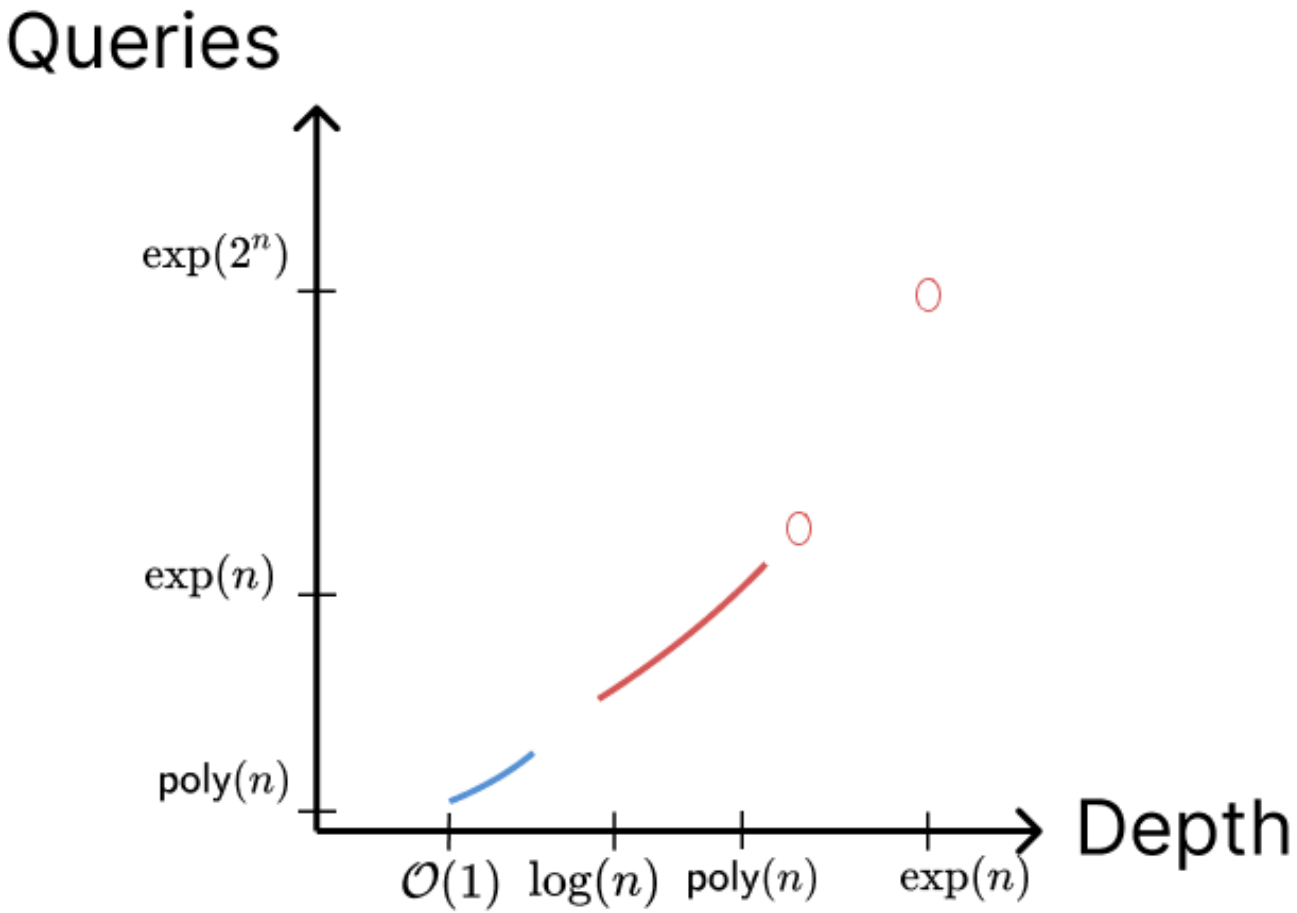

Pseudorandom Unitaries

- We show that pseudorandom unitaries cannot be constructed using $\mathcal{O}(1)$-depth circuits.

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- Recently, [Schuster, Haferkamp & Huang] showed a depth-optimal construction for PRUs in depth $\text{poly}(\log \log n)$.

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- Efficient QStat algorithm for learning local observables.
- Efficient QPStat algorithm for learning shallow quantum circuits.
- Average-case lower bounds for log-depth to linear-depth random quantum circuits.

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- Other interesting setups for learning unknown observables?