On fundamental aspects of Quantum Extreme Learning Machines and Reservoir Computing

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Abstract

Quantum Extreme Learning Machines (QELMs) have emerged as a promising framework for quantum machine learning. Their appeal lies in the rich feature map induced by the dynamics of a quantum substrate -the quantum reservoir- and the efficient post-measurement training via linear regression. Here we study the expressivity of QELMs by decomposing the prediction of QELMs into a Fourier series. We show that the achievable Fourier frequencies are determined by the data encoding scheme, while Fourier coefficients depend on both the reservoir and the measurement. Notably, the expressivity of QELMs is fundamentally limited by the number of Fourier frequencies and the number of observables, while the complexity of the prediction hinges on the reservoir. As a cautionary note on scalability, we identify four sources that can lead to the exponential concentration of the measurement outcomes as the system size grows (Haar-expressivity of both reservoir and encoding, hardware noise, entanglement, and global measurements) and show how this can turn QELMs into useless input-agnostic oracles.

We further generalize our analytics to Quantum Reservoir Computing (QRC), which is typically used for time series prediction. We obtain fundamental upper bounds on the total information processing capacity and show that the above four sources can also induce the exponential concentration of observables for QRC with finite length inputs. Our analysis elucidates the potential and fundamental limitations of QELM and QRC, and lays the groundwork for systematically exploring quantum reservoir systems for other machine learning tasks.

References

[1] Xiong, Weijie, et al. *arXiv preprint arXiv:2312.15124 (2023).*

Figures

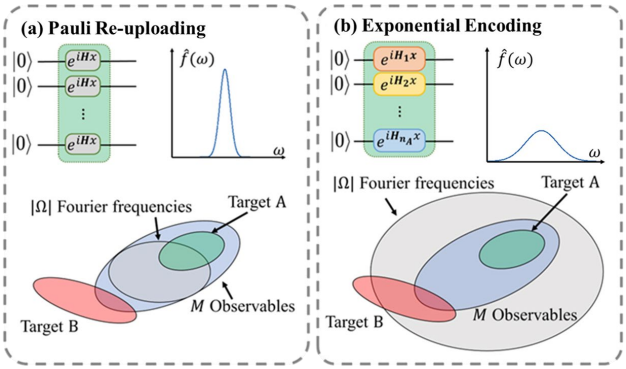

Figure 1: Pauli re-uploading and the exponential encoding lead to polynomially and exponentially many frequencies, resp. Hence, the prediction of a QELM with Pauli encoding has a more concentrated Fourier spectrum, which is more efficiently simulated classically. The exponential encoding, corresponding to the partial control regime M < |Ω|, allows for a wider range of target functions compared to the Pauli re-uploading, where M > |Ω| and might offer a quantum advantage.

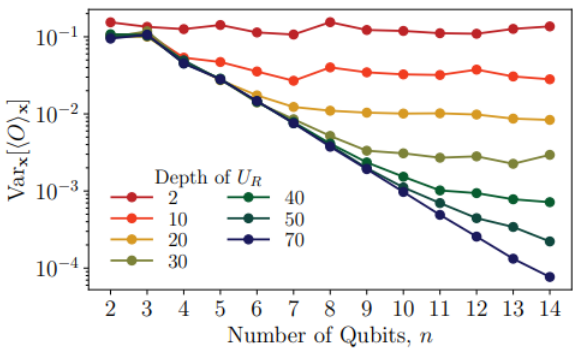

Figure 2: Reservoir Haar-expressivity-induced concentration. Variance of the expectation value of observable Z_1Z_2 over a set of inputs uniformly sampled from [-π, π], as a function of the number of total qubits n and for different depths of the reservoir defined in Eq. (47) of Ref. [1].