(Architectures, random properties and applications of)

Symplectic quantum circuits

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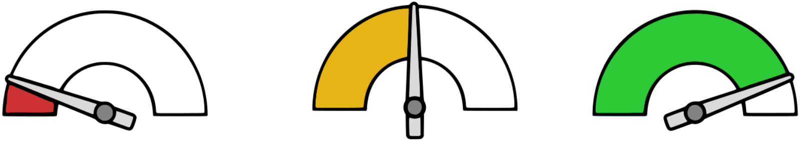
Motivation

Attention received by group:



$\mathbb{SP}(d/2)$ Symplectic	$\mathbb{O}(d)$ Orthogonal	$\mathbb{U}(d)$ Unitary
	$\mathbb{SU}(2)$-equivariant	C_n Clifford
	S_n-equivariant	$\mathbb{SO}(2n)$ Matchgate

Background

Background

- The compact symplectic group: $\mathbb{SP}(d/2) := \mathbb{SP}(d;\mathbb{C}) \cap \mathbb{SU}(d)$
- That is, the group of $d \times d$ unitary symplectic matrices (with d even)
- A symplectic matrix satisfies $S^T \Omega S = \Omega$
- $\Omega = iY \otimes \mathbb{1}_{d/2} = \begin{pmatrix} 0 & \mathbb{1}_{d/2} \\ -\mathbb{1}_{d/2} & 0 \end{pmatrix}$

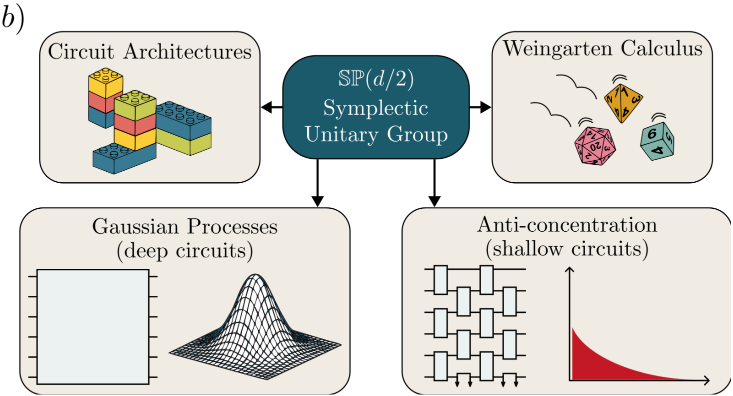
Background

$\mathbb{SP}(d/2)$ is the only subgroup of $\mathbb{SU}(d)$ with pure state controllability:

Any pure state can be reached from any other pure state by a unitary symplectic matrix.

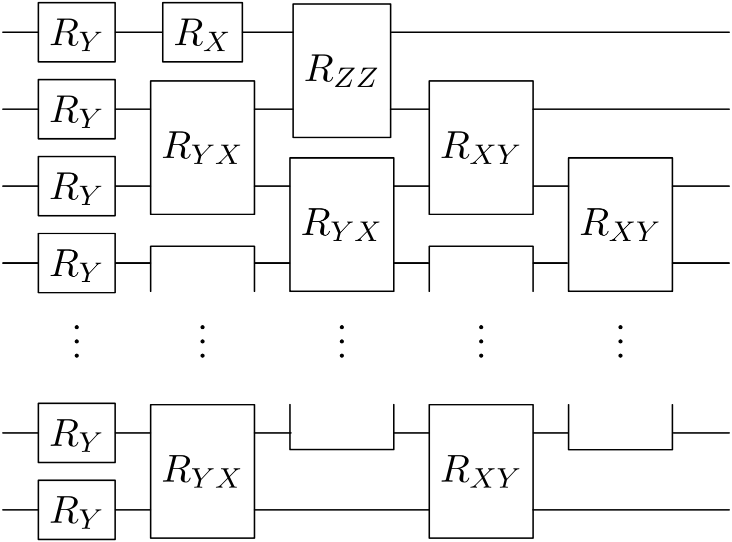
Symplectic quantum circuits

Symplectic quantum circuits

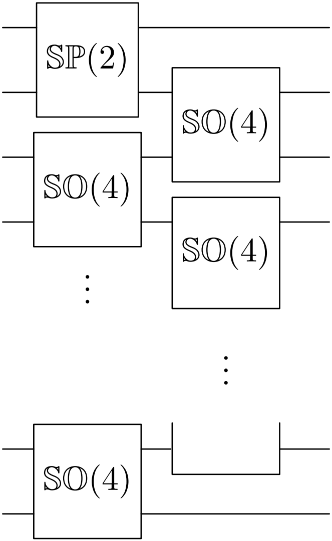


"Architectures and random properties of symplectic quantum circuits"
DGM, P. Braccia & M. Cerezo

Symplectic quantum circuits



Symplectic quantum circuits

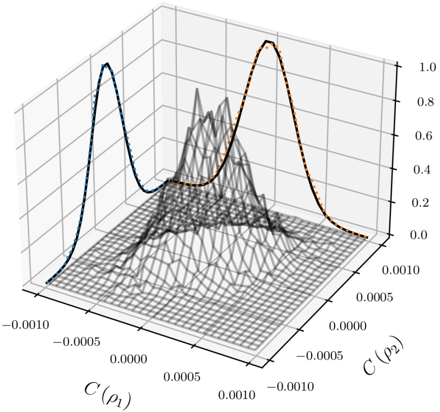


- 1D connectivity
- Generators are **not** translationally invariant!
- Local symplectic unitaries are a universal gate set!

  (Wiersema et al., 2024)

Random properties of symplectic quantum circuits

Symplectic quantum circuits form **Gaussian processes**



- Sample U from $\mathbb{SP}(d/2)$
- Compute $C(\rho_i) = \mathrm{Tr}[U\rho_i U^\dagger O]$
- Plot a histogram

DGM, M. Larocca & M. Cerezo "Quantum neural networks form Gaussian processes"
F. Girardi and G. De Palma "Trained quantum neural networks are Gaussian processes"
E. Anschuetz "A Unified Theory of Quantum Neural Network Loss Landscapes"

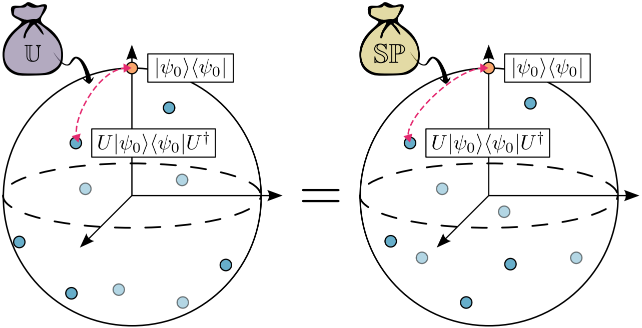
Random properties of symplectic circuits

Teaser: we are **cautiously optimistic** about the possibility of obtaining a quantum advantage with quantum Gaussian process regression.

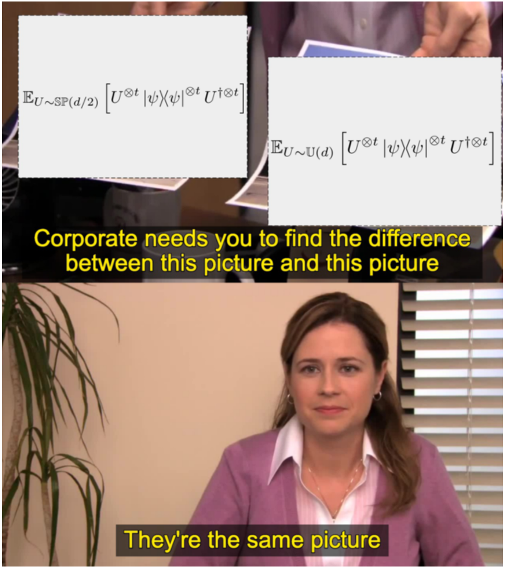
Random properties of symplectic circuits

"Random ensembles of symplectic and unitary states are indistinguishable",

M. West, A. Anna Mele, M. Larocca & M. Cerezo



Random properties of symplectic circuits



Random properties of symplectic circuits

Result 2. *A random circuit in a one-dimensional topology composed of symplectic random two-qubit gates can form an ϵ-approximate 2-design with 60% less parameters than a circuit with the same topology and with unitary random two-qubit gates.*

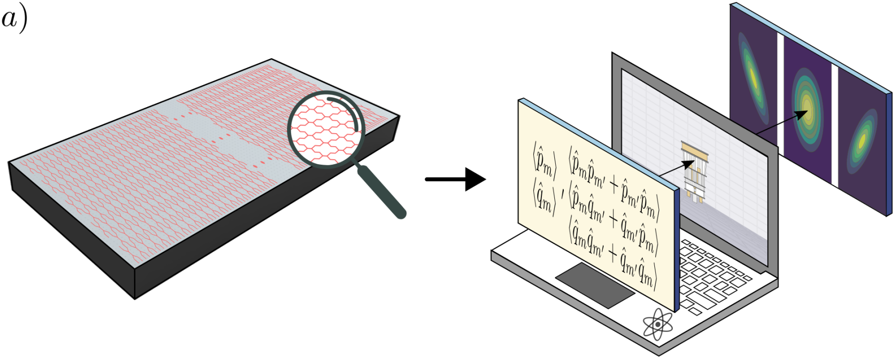
Result 3. *Symplectic classical shadows are equivalent to unitary classical shadows.*

"Random ensembles of symplectic and unitary states are indistinguishable",
M. West, A. Anna Mele, M. Larocca & M. Cerezo

Symplectic quantum simulation

Symplectic quantum simulation

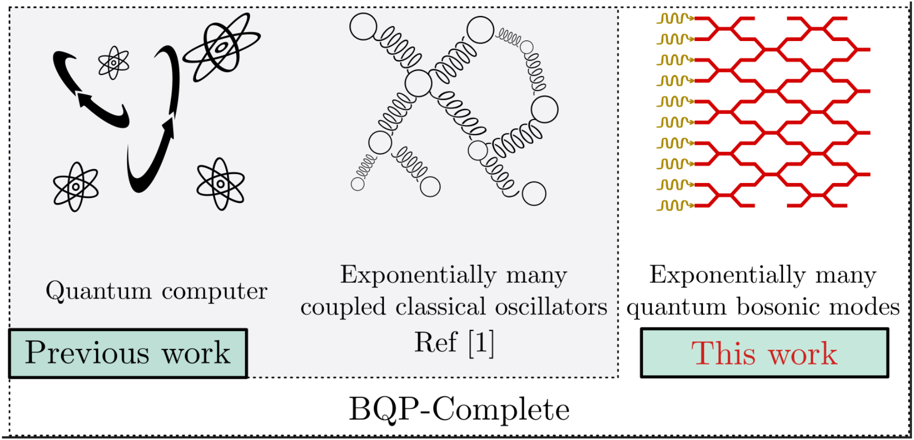
a)



Quantum optics
# of modes $= 2^n$
Gaussian bosonic circuit

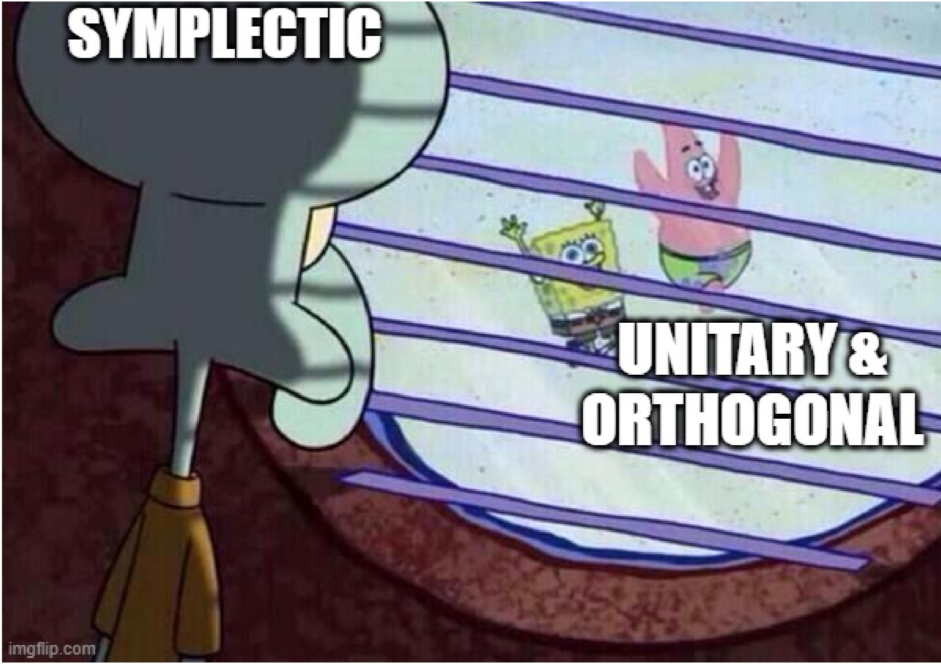
Quantum computer
# of qubits $= n + 1$
Real + Imaginary time evolution

Symplectic quantum simulation



[1] R. Babbush et al. "Exponential quantum speedup in simulating coupled classical oscillators"

Conclusions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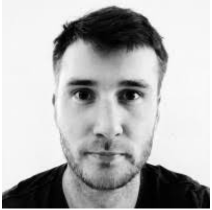
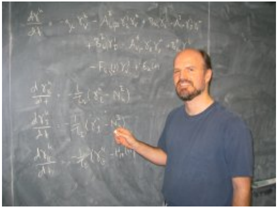

DGM, Paolo Braccia and M. Cerezo (2024)
Architectures and random properties of symplectic quantum circuits arXiv:2405.10264

Alice Barthe, M. Cerezo, Andrew T. Sornborger, Martin Larocca and DGM (2024)
Gate-based quantum simulation of Gaussian bosonic circuits on exponentially many modes arXiv:2407.06290

Maxwell West, Antonio Anna Mele, Martin Larocca and M. Cerezo (2024)
Random ensembles of symplectic and unitary states are indistinguishable arXiv:2409.16500