Information compression via hidden subgroup quantum autoencoders

Feiyang Liu,[1] Kaiming Bian,[1] Fei Meng*,[2,1] Wen Zhang*,[3] and Oscar Dahlsten*[2,1,4]

[1] *Shenzhen Institute for Quantum Science and Engineering and Department of Physics, SUSTech, Nanshan District, Shenzhen, China.*
[2] *Department of Physics, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong SAR*
[3] *HiSilicon Research, Huawei Technologies Co., Ltd., Shenzhen, China*
[4] *Institute of Nanoscience and Applications, Southern University of Science and Technology, Shenzhen 518055, China*

We design a quantum method for classical information compression that exploits the hidden subgroup quantum algorithm. We consider sequence data in a database with *a priori* unknown symmetries of the hidden subgroup type. We show that data with a given group structure can be compressed with the same query complexity as the hidden subgroup problem, which is exponentially faster than the best-known classical algorithms for certain types of HSP problem. We moreover design a quantum algorithm that variationally finds the group structure and uses it to compress the data. There is an encoder and a decoder, along the paradigm of quantum autoencoders. After the training, the encoder outputs a compressed data string and a description of the hidden subgroup symmetry, from which the input data can be recovered by the decoder. In illustrative examples, our algorithm outperforms the classical autoencoder on the mean squared value of test data. This classical-quantum separation in information compression capability has thermodynamical significance: the free energy assigned by a quantum agent to a system can be much higher than that of a classical agent. Taken together, our results show that a possible application of quantum computers is to efficiently compress certain types of data that cannot be efficiently compressed by current methods using classical computers.

I. INTRODUCTION

Information compression is the ubiquitous task of reducing data size by appropriate encoding. A prominent method of compression is to employ autoencoders [1, 2], artificial neural networks that automatically learn to compress unlabeled data without prior knowledge of the underlying patterns. Exploiting a bottleneck structure, an autoencoder can automatically extract essential features as compressed data in such a manner that the original data can be reconstructed [3–5].

Recently, the quantum version of autoencoders [6, 7], has similarly been applied to compress *quantum* data. Quantum autoencoders can, via unitary circuits, compress data hidden in quantum superpositions, but it is natural to wonder if they can also be valuable in compressing *classical* data.

Quantum computers can indeed extract certain features in data that are not efficiently accessible to classical computers. For instance, the quantum period finding algorithm [8–10] is exponentially faster than the best current classical algorithms in identifying the period of a function. More generally, the Abelian hidden subgroup problem (HSP) represents a broad class of problems (e.g., period finding) that do not have known efficient classical algorithms, whereas efficient quantum algorithms often exist [11–15].

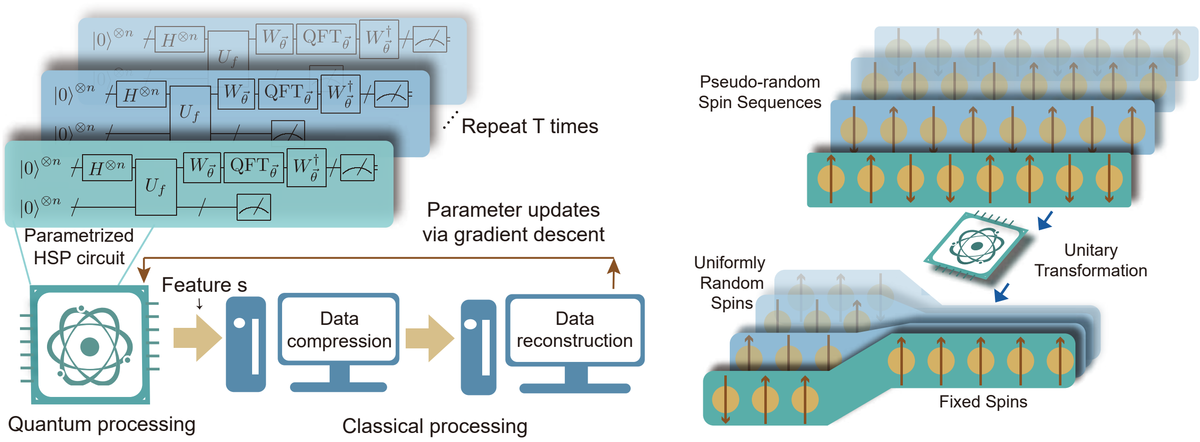

FIG. 1. **The information compression algorithm.** The left panel shows the circuit ansatz and the training scheme. The right panel shows the compression of pseudo-random spin sequences effectively generate fixed spins, which can be used to extract work. Our quantum auto-encoder first applies a parametrized hidden sub-group problem (HSP) circuit using an oracle $U_f |i\rangle |0\rangle = |i\rangle |f(i)\rangle$ and a parametrized quantum Fourier transform $\mathrm{QFT}_{\vec{\theta}}$ for T times to extract the hidden subgroup structure as features s. $W_{\vec{\theta}}$ is a parameterized gate that permutes qubits to search over broader range of group structures. The parameters $\vec{\theta}$ are tuned using gradient descent until the circuit identifies features s representing reversible compressions of the data from the given source.

Can that quantum speedup on HSPs be turned into an advantage for information compression? To tackle this question, we combine quantum autoencoders and the quantum HSP algorithm to create a concrete algorithm to compress data with symmetries of the HSP type.

We prove an exponential speedup in query complexity of quantum algorithms in data compression with symmetries of the hidden subgroup type, extending the quantum computational advantage in HSP to data compression. Then we extend this algorithm to a variational quantum auto-encoder (Fig. 1) by designing a parameterized quantum circuit for HSP, making the variational algorithm capable of finding the hidden subgroup automatically. This is achieved by establishing a parameterized quantum circuit ansatz for quantum Fourier transforms that covers a wide range of the Abelian HSP case. We demonstrate the algorithm explicitly on simple compression examples where classical computers are used to simulate small quantum computers (see Fig. 2).

Our algorithm opens a new direction for quantum machine learning: combining quantum algorithms with quantum autoencoders to achieve more efficient extraction of features. The algorithm exhibits a significant advantage for quantum computers in compressing sequential data over its classical counterpart. Furthermore, this computational advantage in compression can then be mapped to a thermodynamic advantage of energy harvesting, where an intelligent extractor with access to a quantum computer can extract more work within a limited amount of time.

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II. CONCEPTUAL ADVANCEMENTS AND RESULTS.

We consider a database $\mathcal{Q}$ storing sequential data of length $N = 2^n$. The data is represented as an ordered sequence $\{x_0, x_1, ..., x_i, ..., x_{2^n-1}\}$, where each data point $x_i \in \{0,1\}^m$ is a bit string of length m. Using binary representation, we identify any index i and a binary string $\mathbf{i} = i_1 i_2 ... i_n = \tau(i)$. Querying $\mathcal{Q}$ on i yields the value x_i, denoted as $\mathcal{Q}(i) = x_i$.

Assigning group structures to sequential data. We establish group structures for general sequential data, such as time series data, audio streams, video streams, image data, etc. This is accomplished by first describing the index of each data point by a bit string of length n and then assigning Abelian group structures to the set of bit strings of length n. This allows for the exploration of hidden subgroup problems in the application of general sequential data. Our variational algorithms has potential to approximate

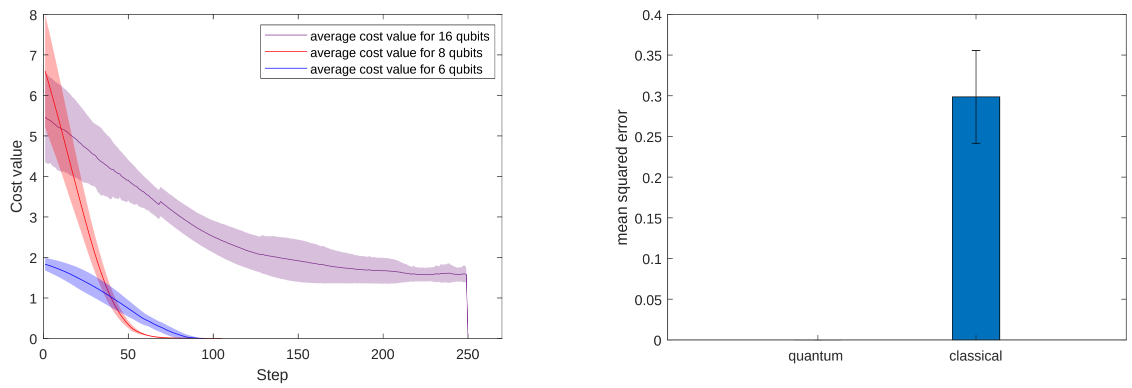

FIG. 2. **Left: convergence of our algorithm under training.** The solid lines are averages and the coloured areas indicate values within one standard deviation of the mean, as obtained from 10 trainings in each case. In one case, $2n = 6$ qubits are used and the data has Simon's symmetry. In a second example, $2n = 8$ qubits are used and the data has periodic symmetry. In the third example, $2n = 16$ qubits are used and the data has Simon's symmetry. The sudden drop at the last step appears when we choose the nearest integer for the generator learned. **Right: comparison of performance of a classical autoencoder and our algorithm.** The performance on test data after training is compared for the quantum case associated with $2n = 16$ qubits, and the classical MATLAB autoencoder[16]. The mean squared error standard deviation over different trainings is also shown as a line segment on the top of the bar. The quantum autoencoder achieves negligible error on the test data, unlike the classical autoencoder.

symmetries in sequential data using hidden subgroups, even if the data has no strict symmetry. This greatly broadens the potential applications of the quantum hidden subgroup algorithm.

HSP compression. Suppose there are many duplicated entries in the database. Then identifying the repeated entries allows for identifying free indices that can be overwritten and thus leads to data compression. Suppose that the generating function f has a hidden subgroup H, i.e. $f(i) = f(j)$ if and only if $i - j \in H$, with respect to a given group structure G on the set of indices $\{i\}_0^{2^n-1}$. A coset c_0H for $c_0 \in G$ is defined as the set $\{c_0 + h | h \in H\}$. Then the hidden subgroup H imposes a hidden pattern in the time series data generated by the function f, which is a redundancy that can be eliminated to achieve data compression. We call this kind of compression *hidden subgroup compression*.

We proved that the query complexity of the HSP compression is the same as the HSP. Then the exponential speedup of quantum algorithms over their classical counterparts in the HSP can be naturally extrapolated to the database compression problem. This is exemplified by the period finding [17, 18] and the Simon's problem [19].

Variational HSP compression. If the time series data has no a priori known group structure, then we need to variationally search over different structures in order to fine one group structure with respect to which the sequential data can be compressed via HSP algorithm. This is achieved by replacing the quantum Fourier transform (QFT) in a standard HSP circuit with a parameterized circuit, which recovers the QFT over any finite Abelian group G based on the chosen parameters. We designed such a parameterized QFT circuit and then incorporated this parametrized hidden subgroup algorithm into an autoencoder structure. The autoencoder compresses the data through its bottleneck using the encoder and restores it using the decoder, ensuring reversible compression. During training, an automatic search finds suitable parameters to make the decoder's output as similar as possible to the input. Once training is complete, the decoder can be removed and the encoder acts as a compressor. This algorithm is illustrated in Fig. 1.

Compression ratio of the encoder. Our algorithm can potentially compress certain types of data with an exponentially decreasing compression ratio. Our algorithm takes $n(n-1)$ bits to specify the group structure. For example, let us consider the subset of all balanced binary sequences of length 2^n, where the sequence has the same number of 0's and 1's, e.g., $\{0, 0, 1, 1\}$. Because 0 and 1 are repeated for 2^{n-1} times, the subgroup H, if it exists, has cardinality 2^{n-1}, which gives us a compression ratio $\kappa = \frac{n+2}{2^n}$, since n bits are needed to specify the generator of H, and 2 bits are used to store the data values $f(c)$ for each coset $c \in G/H$,

Quantum thermodynamical advantage. Our algorithm implies a quantum advantage in energy harvesting. To demonstrate this, we designed an artificial example constructed by adopting Simon's problem [19] into a pseudo-random source: assume that the spin generated at time i will be the same as the spin generated at time j if and only if $i = j \oplus s$ for some unknown parameter s, where $\oplus$ is bit-wise addition modulo 2. For such a correlation pattern, quantum algorithms are proven to be exponentially faster than any classical algorithm [20] in finding out the pattern. At the training stage, our algorithm can be applied to find out the unknown parameter s, and then in the running stage, a unitary transformation can be constructed to convert the non-uniformly distributed long spin sequences into shorter, uniformly random spin sequences, with a fixed number of spins appended at the end, as shown in Fig. 1. From the fixed spins, energy can be extracted using Szilard's engine [21]. Since the quantum HSP algorithm is exponentially faster than any classical algorithm in identifying r, it implies that effectively quantum algorithm can extract more energy from this pseudo-random source within a fixed amount of time.

III. CONCLUSION

We developed a novel quantum algorithm for non-linear data compression, utilizing the hidden subgroup problem (HSP) and extending it to a self-adjusting variational version. This approach uses a versatile quantum circuit ansatz for diverse Abelian HSP scenarios, permitting selective feature extraction for compression. Our HSP-based compression algorithm shows scalable performance and convergence in the numerical simulations of error-corrected quantum computers.

Our work bridges quantum HSP exponential speed-up with data compression, offering a promising new research direction into quantum autoencoders and HSP, and opening avenues for future explorations in the non-Abelian HSP context, making a significant stride in the evolution of quantum computing and machine learning. Full version of the paper can be found on arXiv [22].

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