Title: Adaptive Quantum magnetometry using Myopic Entropy reduction

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Introduction:

Adaptive sensing is an iterative approach in which measurement methods are changed dynamically in response to previous measurement outcomes. Adaptive sensing offers to improve the sensitivity scaling of quantum magnetometry by a factor of √N , for N measurements comparing to constant parameter sensing. This improved scaling saturates the Heisenberg limit, offering optimal information gain. Implementing adaptive sensing in the context of NV magnetometry can be difficult from a practical perspective, due to measurement fidelity and computation time requirements typical of high-speed Ramsey experiments. We introduce a myopic entropy reduction approach within a Bayesian framework, which can be tuned to specific measurement setups. In the absence of bias decoherence and prior information, we show that following the myopic approach is equivalent to the Kitaev phase estimation (KPE) algorithm. We also demonstrate a limitation of the KPE algorithm when implemented on a Gaussian prior and compare the Myopic approach. We present a conceptual framework for interpreting the process of optimising posterior entropy, based on mutual information and likelihood functions. This approach offers further generalisations to the KPE algorithm tailored to experimentally practical magnetometry.

Framework and methods:

We adopt a Bayesian approach to simulating and updating our estimation of the probability, through the form of a posterior distribution. We use the following form for the likelihood of the Ramsey experiment,

$$\Pr(X=0|B,\tau,\theta)=\frac{1+\mathrm{e}^{-\frac{\tau}{T}}\cos(2\mu B\tau+\theta)}{2} \tag{1}$$

with phase θ, and exposure time τ, as tuneable parameters, magnetic field B, decoherence T, and coupling strength μ. We leverage the periodic behaviour exhibited by the Ramsey experiment, allowing for significant analytic simplifications based on the Fourier transform. We can quantify the gain of information throughout the measurement process using the Shannon information over the posterior distribution. We chose measurement parameters by maximising the expected reduction in the Shannon entropy of the posterior distribution post-measurement.

Results:

We have simulated the performance of myopic optimisation algorithm and compared it to KPE and random sampling, demonstrating the limiting behaviour of KPE on a Gaussian prior and the effectiveness of the myopic approach for entropy and variance reduction (see Fig.1). We have shown analytically that given idealised measurement conditions (i.e., in the absence measurement bias or decoherence), myopic optimisation over the diffuse prior is equivalent to the KPE algorithm. The analytic process required to derive entropy reduction creates qualitative descriptions of measurement entropy and conditional entropy in terms of measurement properties of the likelihood function.

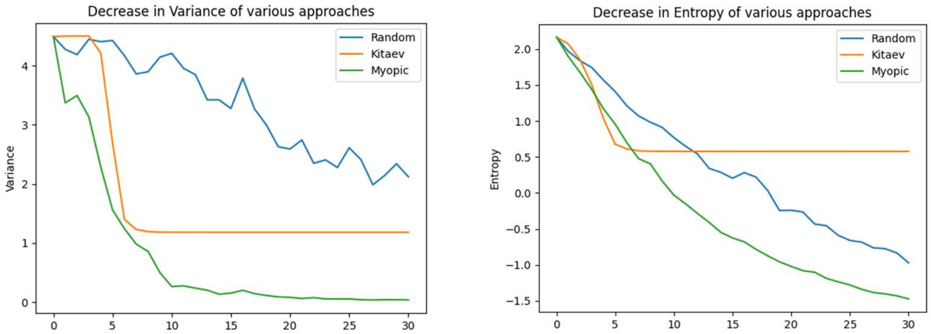

Figure 1: The decrease in posterior variance (left) and entropy (right) after each measurement, starting with a Gaussian prior, using KPE (orange), myopic (green) and random (blue) algorithms to generate measurement parameters.

Significance:

The myopic approach can be applied to likelihood functions beyond the simplified model shown by Eq.1.This means, unlike the KPE algorithm, the myopic approach generalises to a wide set of measurement models and can be tailored for individual experimental implementations. The submodularity of Shannon entropy guarantees scaling that is optimal up to a constant term. The lessons learnt in posterior optimisation can be applied more generally to other contexts so long as a likelihood function can be constructed.