Advantages of quantum support vector machine in cross-domain classification of quantum states

Diksha Sharma[1], Vivek Balasaheb Sabale[1*], Parvinder Singh[2], and Atul Kumar [1]
[1]*Indian Institute of Technology Jodhpur, Rajasthan-342037, India*
[2]*Central University of Punjab Bathinda, Punjab-151001, India*
[*]sabale.1@iitj.ac.in

Abstract

In this study, we use the cross-domain classification using quantum machine learning for quantum advantages to address the entanglement versus separability paradigm. The inherited structure of quantum states and its relation with a particular class of quantum states are exploited to intuitively approach the classification of different domain testing states, referred here *cross-domain classification*. In addition, we extend our analysis to evaluate the robustness of our model for the analyzed problem using random unitary transformations. Using numerical analysis, our results clearly demonstrate the potential of QSVM for classifying quantum states across the multi-dimensional Hilbert space. We further extend the study for efficient classification of Bell diagonal states into zero and non-zero discord classes.

Keywords: Quantum Machine Learning, Entanglement classification, Cross-domain classification, Support Vector Machine.

1 Introduction and Proposed Approach

In general, quantum states are categorized into an entangled class or a separable class. The entangled versus separability paradigm has led the debate for achieving quantum advantages using entangled resources in comparison to the use of classical resources [1, 2, 3, 4, 5]. Therefore, the classification of quantum states into an entangled or separable class is both a fundamental and application-oriented problem. However, as the number of qubits in a quantum system increases, the number of classes also increases. For example, bipartite or two-qubit quantum states are typically classified into two potential classes, while this classification expands to five and nine classes for three and four-qubit quantum systems [6, 7]. One of the efficient ways to address the entanglement versus separability paradigm is to approach the problem through machine learning algorithms as a pattern recognition task. In the conventional machine learning methods, a super-set comprising identically distributed states serves as the training set and a subset of this super-set is designated as the testing set. In this work, we readdress the entanglement versus separability problem using quantum machine learning algorithms. For this, we use one set of states as training sets in a specific Hilbert space to effectively classify a spatially different set of quantum states in a different Hilbert space- *Cross-domain classification*. The inherent mathematical structure of quantum states provides a basis for such a *cross-domain classification* [8, 9] and further assists in reducing the training space. We propose a quantum machine learning algorithm, namely, a quantum support vector machine (QSVM) [10, 11] that employs the benefits of recognizing patterns in quantum states more effectively than classical machine learning algorithms.

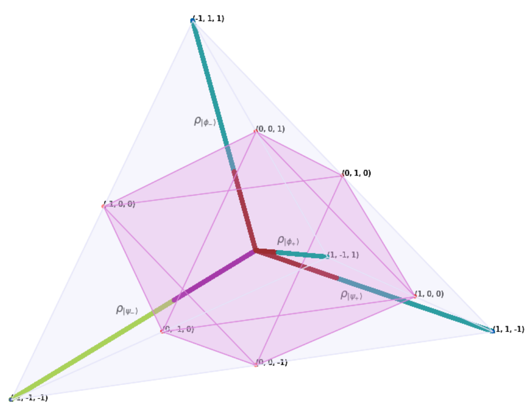


Figure 1: *Geometrical representation of training and testing two-qubit Werner states; the four corners represent Bell states such that (-1,-1,-1), (-1,1,1), (1,-1,1), and (1,1,-1) represent $|\psi_-\rangle$, $|\phi_-\rangle$, $|\phi_+\rangle$, and $|\psi_+\rangle$, respectively. The line joining from the center to the vertex represents one of the Bell diagonal states, and the part of the line confined in the octahedron depicts a separable class of quantum states.*

We first develop models for different strategies for two-qubit states to analyze the robustness of the models in classifying separable and entangled quantum states. Specifically for cross-domain classification, we present the analysis of our model (QSVM) for two-qubit Werner states [12]. The trained model for a two-qubit Werner state is then used for the classification of Horodecki and maximally entangled mixed states (MEMS) [13, 14, 15, 16]. The difference in training and testing states- *cross domain classification*- is visualized for the case of Bell diagonal states [17] using three diagonal elements of the correlation matrix (Refer to Eq.(1)). The Bell diagonal states are of the following form

$$\rho_{BD} = \frac{1}{4}(I \otimes I + \sum_{i=1}^{3} t_{ii}\sigma_i \otimes \sigma_i), \tag{1}$$

where elements of correlation matrix t_{ii} ranges from -1 to $+1$. Fig. (1) represents the visualization for a specific case of Bell diagonal states, namely, the Werner class of states where Werner states are two-qubit mixed states expressed as

$$\rho_{|\psi\rangle} = p\,|\psi\rangle\,\langle\psi| + \frac{(1-p)}{4}I, \tag{2}$$

here $|\psi\rangle$ represents one of the Bell states given as $|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}\,|00\rangle \pm |11\rangle$ or $|\phi_{\pm}\rangle = \frac{1}{\sqrt{2}}\,|01\rangle \pm |10\rangle$ and p represents the probability. It is well established that the Werner states are entangled for $p > \frac{1}{3}$ and separable for $p \leq \frac{1}{3}$. The states with $p \leq \frac{1}{3}$, though separable, still exhibit non-classical correlations useful for quantum information and computation [17]. These properties of Werner states serve as key ingredients to analyze them to understand foundational aspects of quantum theory. Clearly, Werner states contain separable as well as entangled states with the complete class being useful for applications in quantum information and computation due to nonlocal or non-classical correlations. For convenience, the four types of Werner states are depicted in Fig. 1 highlighting their similarities and differences in their form as represented in Eq. (2). We further perform a comprehensive analysis to study and evaluate the robustness of our model using random unitary operations. The performance of the algorithm is evaluated using evaluation metrics such as accuracy, precision, recall, and F1-score.

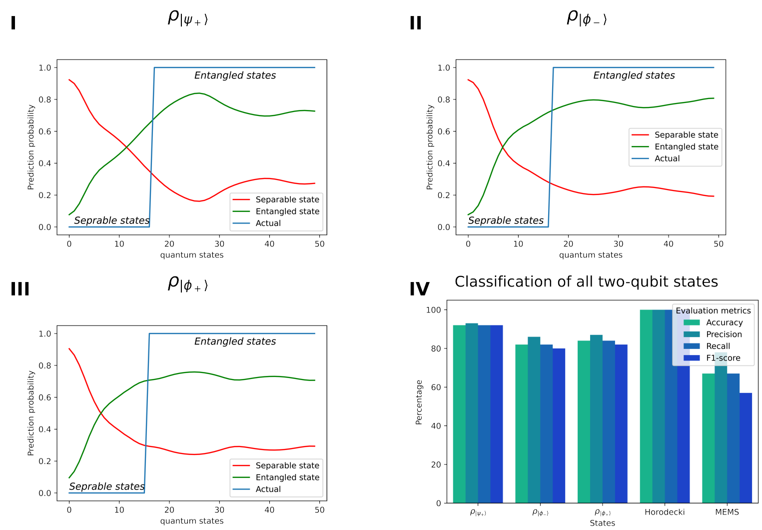
2 Results

In this section, we proceed to analyze the advantages of QSVM [18] in comparison to classical support vector machine (CSVM) [19] for employing cross-domain classification to entanglement versus separability problem. Fig. 2A clearly demonstrates the advantages of using QSVM for classifying the Werner, Horodecki, and MEMS states in entangled or separable states. For example, Figs. 2A- I, II and III represent the QSVM predictions corresponding to $\rho_{|\psi_+\rangle}$, $\rho_{|\phi_-\rangle}$ and $\rho_{|\phi_+\rangle}$, respectively when the model is trained using a Hilbert space occupied by the $\rho_{|\psi_-\rangle}$ states. The figures illustrate that if the Werner state is separable, then the prediction probability associated with separable states (the red line) needs to be considerably higher than the prediction probability associated with entangled states (the green line) which is in concurrence with our results in Figs. 2A. Further, the actual class of a particular quantum state is depicted through the blue line. Therefore, Fig. 2A-I shows the prediction probability corresponding to individual quantum states belonging to $\rho_{|\psi_+\rangle}$. The figures corresponding to $\rho_{|\phi_-\rangle}$ and $\rho_{|\phi_+\rangle}$ represent similar predictions. Our results show that the proposed model can predict the separable and entangled states with a significant probability. The same can be inferred from plots of evaluation metrics, i.e., accuracy, precision, recall, and F1-score as represented in Fig. 2A-IV. Hence, QSVM successfully performs even the *cross-domain classification* to address entanglement versus separability problem.

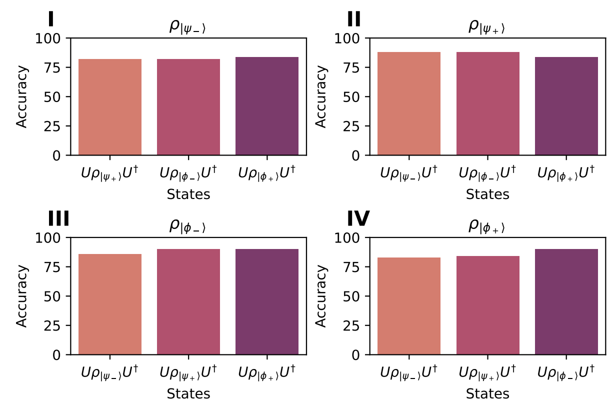
For a comprehensive analysis of two-qubit quantum states under *cross-domain classification* strategy, we further evaluate our models on Horodecki and MEMS states while training the model on the Werner state ($\rho_{|\psi_-\rangle}$). Fig. 2B illustrates the result of predicting the Horodecki and MEMS states as entangled or separable using QSVM. Interestingly, for Horodecki states, QSVM shows a 100% accuracy. For MEMS states, the quantum support vector machine shows a moderate accuracy compared to Wener and Horodecki states. This can be attributed to the significant difference between the $\rho_{|\psi_-\rangle}$ state and MEMS states. For comparative analysis, we also evaluate the prediction capability of CSVM and artificial neural network (ANN) under *cross-domain classification* strategy. Surprisingly, we find that both CSVM and ANN models predict each set of testing states as separable states.

2.1 Analysing robustness of QSVM

In order to further ascertain the predictive capacity of the QSVM on Werner states, we introduce variability for testing the trained model by evaluating it on Werner states transformed by local unitary operation. Fig. 2B represents the accuracy corresponding to different Werner-type states. Our analysis reveals that the model consistently achieves superior accuracy in predicting entangled and separable states, even in the case of random unitary transformations. This analysis rigorously assesses the capability of QSVM for *cross-domain classification* of quantum states. For further comparison, one may compare Figs. 2A and 2B.



A. predictions by the model on quantum states, trained on $\rho_{|\psi_-\rangle}$



B. predictions by the model on the random unitary transformation of Werner states

Figure 2: *The analysis of quantum support vector machine. In 2A-I, 2A-II and 2A-III, the model is trained on* $\rho_{|\psi_-\rangle}$ *and tested on* $\rho_{|\psi_+\rangle}$, $\rho_{|\phi_-\rangle}$, *and* $\rho_{|\phi_+\rangle}$, *respectively on basis of prediction probability corresponding to each testing quantum states. 2A-IV includes the analysis of Werner states along with Horodecki and MEMS states based on evaluation metrics. B) Accuracy-based analysis of quantum support vector machine for unitary transformation of Werner states.*

3 Conclusion

This study delved into the efficacy of employing a quantum support vector machine (QSVM) to classify quantum states. By tapping into the inherent structure within quantum states of the same family, we aimed to streamline the task of entanglement detection, identification, or classification. We specifically emphasized on an efficient technique called cross-domain classification to classify the unknown quantum states. One can envision the distinct domains of both training and testing states through the geometrical representation used in this study. To validate our approach, we conducted experiments across various scenarios. Initially, we trained our QSVM model on one variant of Werner states ($\rho_{|\psi_-\rangle}$), and then tested it on different types of Werner states ($\rho_{|\psi_+\rangle}$, $\rho_{|\phi_+\rangle}$, and $\rho_{|\phi_-\rangle}$). Additionally, we included Horodecki and MEMS states in our testing dataset to comprehensively evaluate cross-domain classification capabilities of our model. Our results indicate that the model effectively predicted entangled and separable states with significant accuracy. Furthermore, applying random unitary transformations to quantum states yielded comparable accuracy, reinforcing the robustness of our approach. We extended our cross-domain classification method to further classify non-zero and zero discord states, where we restricted the range of correlation matrix elements for creating the training and testing states, and we also used the same training states for analyzing Werner states as non-zero discord states. We conclusively demonstrated that the QSVM can detect entanglement and quantum correlation without even getting a similar domain of training states. We found that the introduction of cross-domain classification in entanglement detection problems reduces the training states, making the quantum machine learning algorithms more efficient. This reduction in training data not only aids in computational efficiency but also contributes to shorter computing times, making the approach more practical and scalable.

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