Stabilizer Tensor Networks with Magic State Injection

A. C. Nakhl[1]*, B. Harper[1,2], M. West[1], N. Dowling[3], M. Sevior[1], T. Quella[4], M. Usman[1,2,3]

[1]School of Physics, The University of Melbourne, Parkville, 3010, VIC, Australia

[2]Data61, CSIRO, Clayton, 3168, VIC, Australia

[3]School of Physics & Astronomy, Monash University, Clayton, VIC 3800, Australia

[4]School of Mathematics and Statistics, The University of Melbourne, Parkville, 3010, VIC, Australia

*a.nakhl@student.unimelb.edu.au

IBM Quantum Network Hub at the University of Melbourne

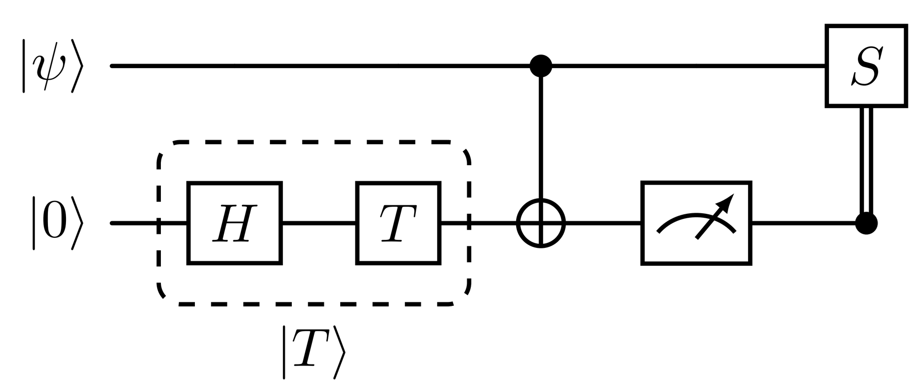
arxiv:2411.12482

Abstract

The Stabilizer Tensor Network (STN) protocol provides an intriguing new simulation paradigm, combining the advantages of both Stabilizer Tableau and matrix-product-state (MPS) methods. We augment this method with magic state injection, reporting a new framework with significantly enhanced ability to simulate circuits with an extensive number of non-Clifford operations. Specifically, for random T-doped N-qubit Clifford circuits the computational cost of circuits prepared with magic state injection scales as $\mathcal{O}(\text{poly}(N))$ when the circuit has $t \lesssim N$ T-gates compared to an exponential scaling for the STN approach, which is demonstrated in systems of up to 200 qubits. In the case of the Hidden Bit Shift circuit, a paradigmatic benchmarking system for extended stabilizer methods with a tunable amount of magic, we report that our magic state injected STN framework can efficiently simulate 4000 qubits and 320 T-gates.

Quantum Computing with MAST

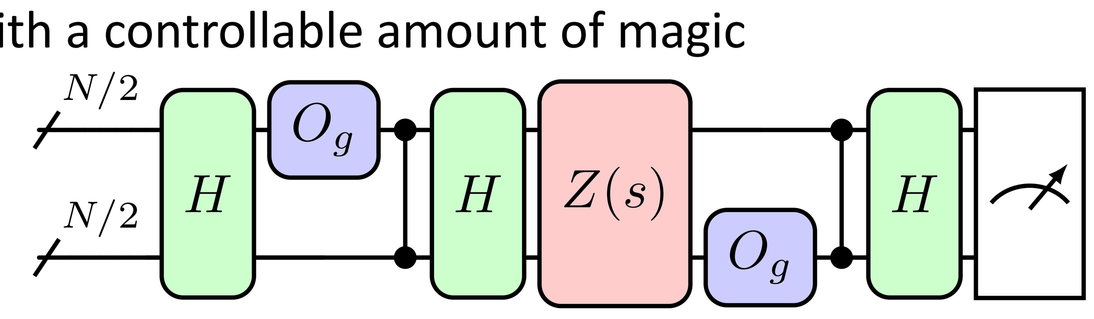
- Magic State Injection Augmented Stabilizer Tensor Networks (MAST) [1] is an augmentation of the Stabilizer Tensor Network (STN) simulation method [2].
- STNs are hybrid stabilizer, tensor network simulation method that can simulate highly entangled states with low magic, and highly magical states with low entanglement.
- Magic State Injection is a gadget used in quantum error correction to perform non-Clifford gates using only Cliffords on the data qubits.



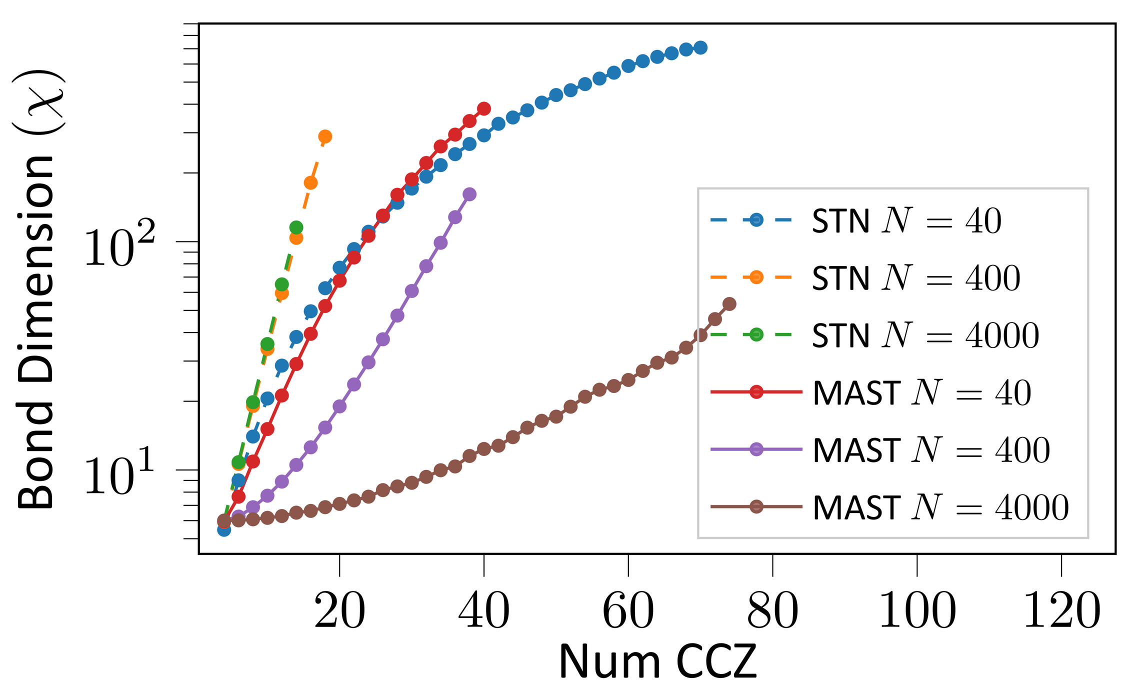
- By using Magic State Injection one can push all the complexity of the simulation to the projective measurement step of the ancilla qubits.

Hidden Bit Shift Circuit

- The Hidden Bit Shift circuit is a paradigmatic circuit for benchmarking circuits with a controllable amount of magic



- There are two different CCZ decompositions considered, one that requires 7 T-gates and and one that requires only 4 T-gates but requires two ancilla qubits.
- In the limit of large N, MAST outperforms STNs, however both scale exponentially with increasing non-Clifford operations.
- For fixed magic, increasing system size reduces simulation cost.
- MAST scales similarly regardless of the CCZ decomposition used unlike STNs (see Figure 3 of [1]).



Hidden Bit Shift with 7 T-gate CCZ decomposition

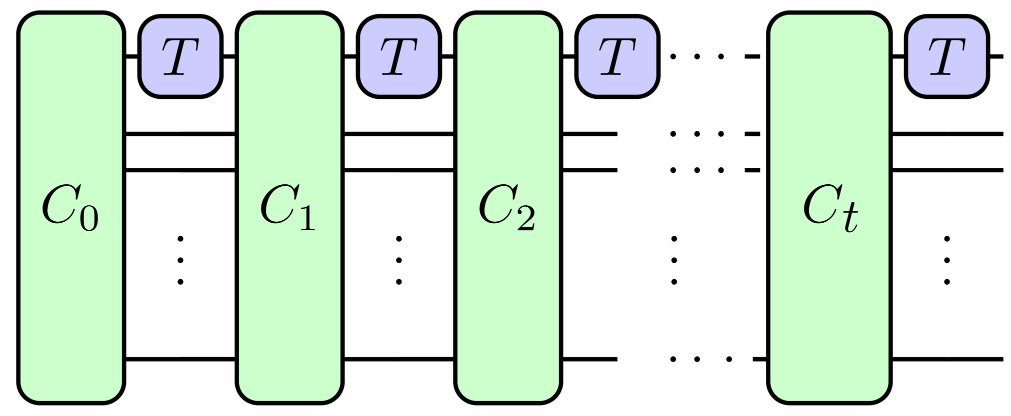
References

[1] Nakhl, A.C., Harper. B, West. M, Dowling. N, Sevior. M, Quella, T. and Usman, M. Stabilizer Tensor Networks with Magic State Injection. arxiv:2411.12482

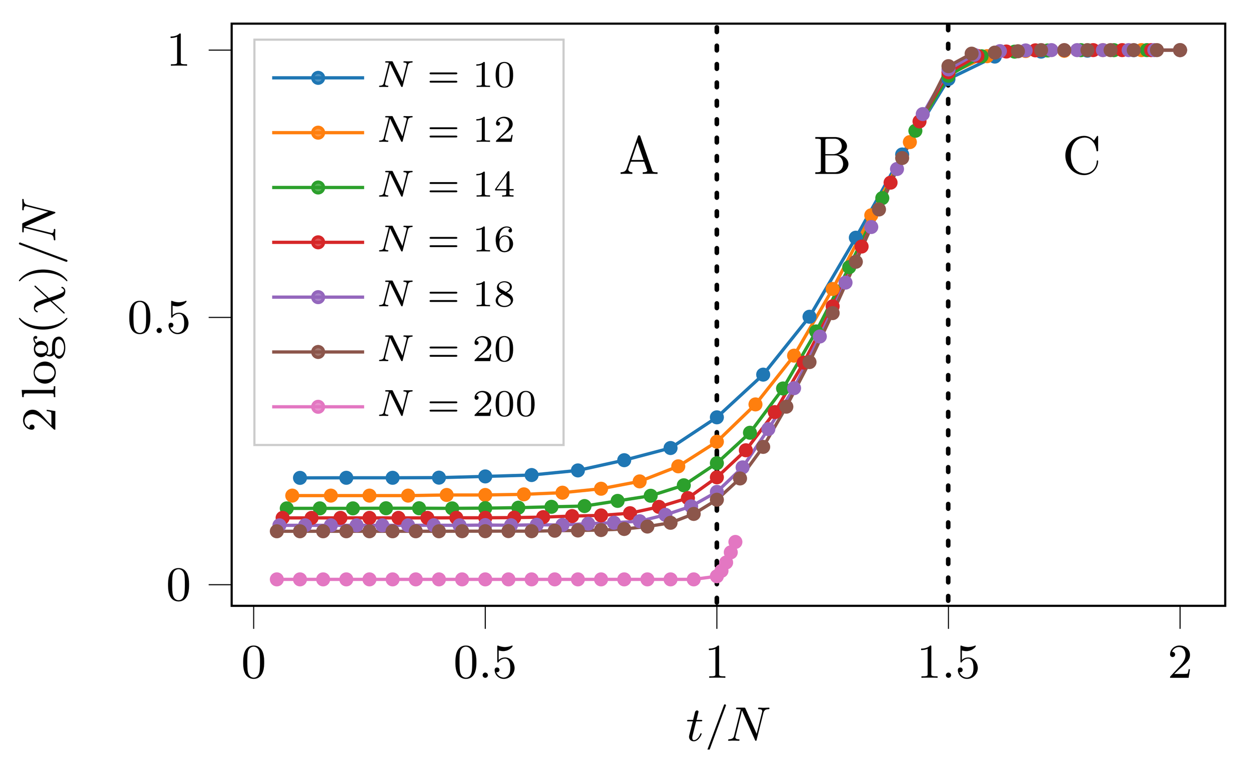
[2] Masot-Llima. S, Garcia-Saez. A. Stabilizer Tensor Networks: universal quantum simulator on a basis of stabilizer states, arxiv:2403.08724

T-doped Clifford circuits

- We simulate random N-qubit T-doped Cliffords of the form

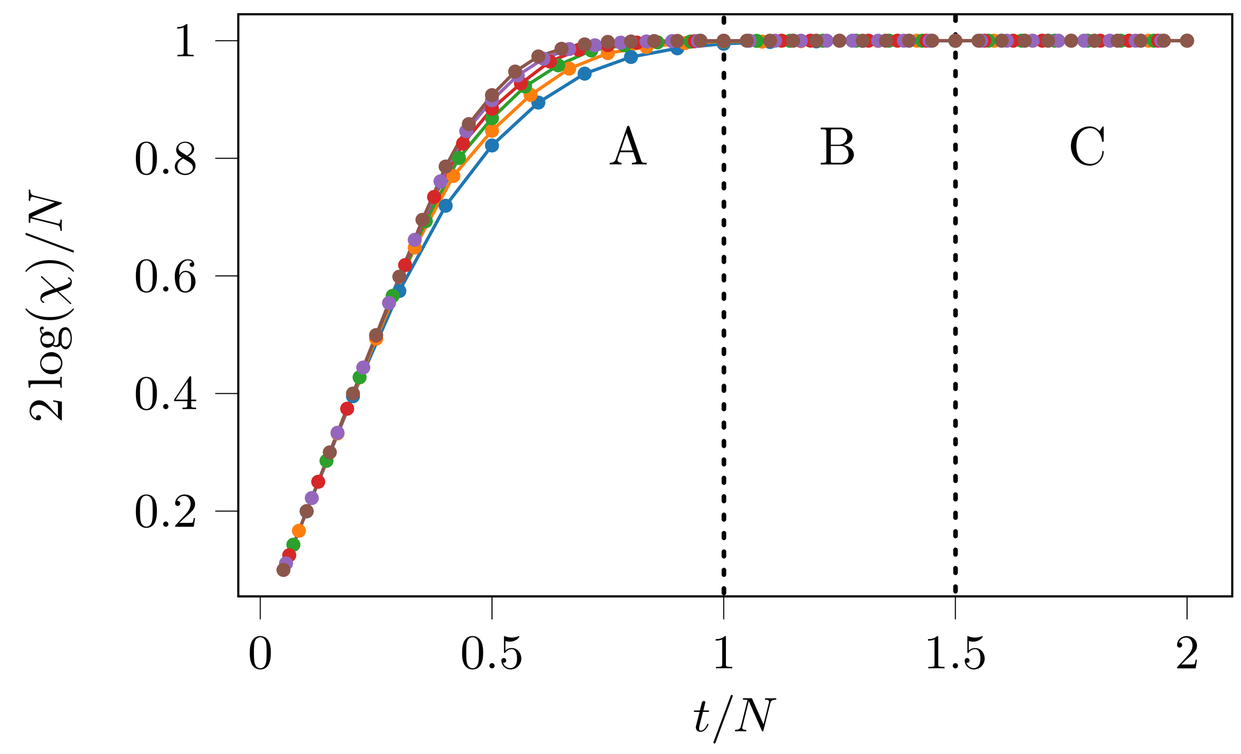


- With MAST the bond-dimension χ of these circuits is bounded by 3 for $t \lesssim N$ (Region A)
- For the intermediate-depth case (Region B) where $N \lesssim t \lesssim 1.5N$ there is an exponential increase in bond-dimension, saturating at $\chi = 2^{N/2}$



Bond-dimension of random T-doped Clifford circuits with MAST

- In comparison, for standard STNs the bond-dimension increases exponentially in Region A, and is saturated in regions B, C.



Bond-dimension of random T-doped Clifford circuits with STN

- We also observe a path dependence with respect to the order that the ancilla qubits are projected — this can most explicitly be seen for simulations of $UU^\dagger |0\rangle$.

Some Open Questions

- How precisely does the resource cost of the simulation relate to the entanglement and magic of the system being simulated?
- Can MAST or STN be further refined by decomposing multi-qubit non-Clifford gates more efficiently?