Calibrating the role of entanglement in variational quantum circuits

Azar C. Nakhl[1], Thomas Quella[2], Muhammad Usman[1,3]

[1] *School of Physics, The University of Melbourne, Parkville, 3010, VIC, Australia*
[2] *School of Mathematics and Statistics, The University of Melbourne, Parkville, 3010, VIC, Australia*
[3] *Data61, CSIRO, Clayton, 3168, VIC, Australia*

Email: a.nakhl@student.unimelb.edu.au

Entanglement is a key property of quantum computing that separates it from its classical counterpart, however, its exact role in the performance of quantum algorithms, especially Variational Quantum Algorithms (VQAs), is not well understood. Unlike conventional quantum algorithms, VQAs rely on heuristic approaches, meaning that even without noise there is no guarantee of these algorithms' performance. As such, entanglement is a key metric in evaluating the performance of these algorithms and helps shed light as to whether these algorithms may be executed successfully on NISQ-era devices.

Tensor Networks have found significant utility over recent years for their ability to simulate slightly entangled quantum systems in a way that is memory-efficient[1]. In addition to this, this simulation method also provides a way to approximate quantum states by enforcing a limitation in the entanglement of the state. This provides a way to probe entanglement in quantum algorithms beyond simply measuring the entanglement entropy throughout the evolution of the state.

In our work[2], we utilise tensor network methods to systematically probe the role of entanglement in the working of two variational quantum algorithms, the Quantum Approximate Optimisation Algorithm (QAOA)[3] and Quantum Neural Networks (QNNs)[4], on prototypical problems under controlled entanglement environments. We find that for the MAX-CUT problem solved using QAOA, the fidelity as a function of entanglement is highly dependent on the number of layers, layout of edges in the graph, and edge density, generally exhibiting that a high number of layers indicates a higher resilience to truncation of entanglement. This is in contrast to previous studies[5] based on no more than four QAOA layers which show that the fidelity of QAOA follows a scaling law with respect to the entanglement per qubit of the system. This suggests that for certain classes of problems, QAOA may be successfully executed on quantum devices that are incapable of generating highly entangled states.

Contrarily, in the case of QNNs, circuits trained to classify images in the standard image datasets (MNIST, FMNIST and CIFAR) to high accuracy are underpinned by higher entanglement, as seen in Figure 1(a) with any enforced limitation in entanglement resulting in a sharp decline in test accuracy. This is corroborated by the entanglement entropy of these circuits which is consistently high suggesting that QNNs require quantum devices capable of generating highly entangled states. We find little dependence on the circuit ansatz, except in so far as some ansätze generate entanglement more slowly as a function of circuit depth. The complexity of the dataset, although having a significant effect on the depth required to achieve high test accuracy did not result in models which are inherently more resilient to enforced limitations in entanglement.

Additionally, we considered the change in entanglement entropy throughout trained QNN circuits as per Figure 1(b). We find for deeper models that the entanglement entropy reduced slightly towards the end of the circuit, indicating that deeper models perform classification in such a way that maximises the probability of the same classification being made for any given input. On the other hand though, as entanglement only drops off at the end of the circuit, regardless of circuit depth, it suggests that training QNN models layer by layer would not be successful as the entanglement of a model at n layers is noticeably different for a model with n layers compared to a model with $N > n$ layers.

Overall our work provides a deeper understanding of the role of entanglement in the working of variational quantum algorithms. In the current NISQ era of quantum computing where noise or errors in quantum devices limit the entanglement that can be generated in quantum circuits, our work will enable their implementations with optimal accuracy within the constraint of affordable entanglement.

References

[1] Bañuls, M. C. et al. "Simulation of many-qubit quantum computation with matrix product states". In: *Physical Review A* 73.2 (Feb. 2006), p. 022344. ISSN: 1050-2947, 1094-1622. DOI: 10.1103/PhysRevA.73.022344.

[2] Nakhl, A. C. et al. "Calibrating the role of entanglement in variational quantum circuits". In: *Physical Review A* 109.3 (2024), p. 032413.

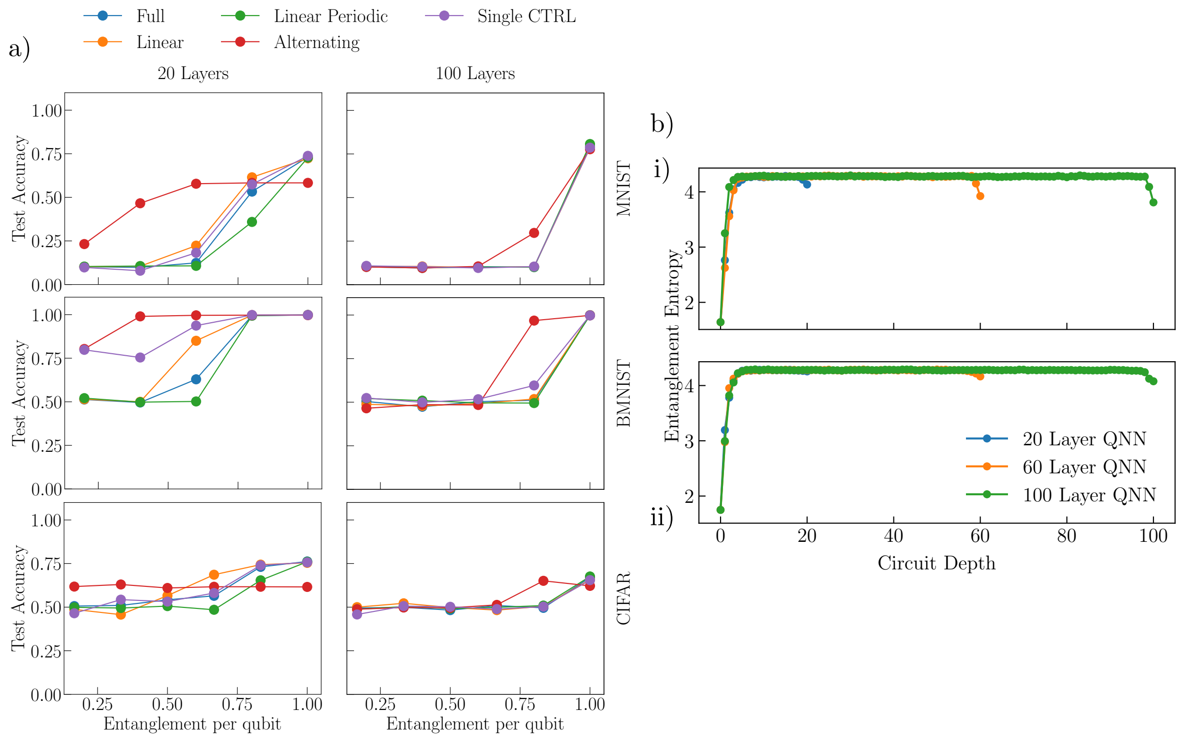

Figure 1: a) Test accuracy for various datasets under limited entanglement simulations. The various circuit ansatz are defined in Figure 1 of Ref. [2]. Note that deeper QNNs are more able to utilise entanglement in a useful manner. We recognise that this is not necessarily a result of shallower circuits being less entangled as shallower circuits are found to have as high an entanglement entropy for QNNs above 20 layer b) The evolution of entanglement throughout the circuit for 20, 60 and 100 layer periodic ansatz QNNs trained on i) the binary MNIST dataset and ii) the 10 class MNIST dataset. Test accuracies for i) are 0.998, 0.998 and 0.999 respectively and 0.728, 0.774 and 0.807 for ii).

[3] Farhi, E. et al. *A Quantum Approximate Optimization Algorithm.* arXiv:1411.4028 [quant-ph]. Nov. 2014.

[4] Farhi, E. et al. *Classification with Quantum Neural Networks on Near Term Processors.* arXiv:1802.06002 [quant-ph]. Aug. 2018.

[5] Dupont, M. et al. "Calibrating the Classical Hardness of the Quantum Approximate Optimization Algorithm". In: *PRX Quantum* 3.4 (Dec. 2022), p. 040339. ISSN: 2691-3399. DOI: 10.1103/PRXQuantum.3.040339.